Ranking Indices for Mitigating Project Risks

Stefan Creemers
Stijn Van de Vonder
Erik Demeulemeester

Abstract - **The goal of project risk management is to mitigate the impact of risks on project objectives such as budget and time. A popular approach to determine where to focus mitigation efforts, is the use of so-called "ranking indices". Ranking indices produce a ranking of activities (or even better, risks) based on their impact on project objectives. In turn, this ranking can be used to determine the risks that are to be mitigated. Different ranking indices, however, produce different rankings. Therefore, one might wonder which ranking index is best? In this article, we provide an answer to this question.**

Keywords - **project risk, ranking indices, risk management, risk analysis, risk mitigation**

1 Introduction

A recent study shows that projects worldwide are still struggling to meet their objectives (Standish Group 2009). During project execution, unforeseen events occur that disrupt plans and that give rise to substantial budget overruns. Risk management is widely recognized as an essential tool to deal with this kind of project uncertainty (see for instance Chap. 60 of this handbook).

The Project Management Institute (PMI 2008) defines risk management as the process that deals with the planning, identification, analyzing, responding, monitoring, and controlling of project risks. In this article, we focus on the risk analysis process and its effect on the risk response process. The risk analysis process can be divided into three subprocesses: risk prioritization, quantitative risk assessment, and quantitative risk evaluation. Risk prioritization is a qualitative procedure that allows to prioritize the risks that have been identified in an earlier stage of the risk management process. Using ordinal estimates of both probability of occurrence and impact of a risk, a shortlist of high-priority risks can be created. During risk assessment, experts provide detailed estimates of the probability of occurrence and the impact of high-priority risks. These estimates are used in the quantitative risk evaluation procedure to analyze the impact of the shortlisted risks on overall project objectives. Fig. 1 provides an overview of the risk analysis process.

Good risk management requires a risk analysis process that is scientifically sound and that is supported by quantitative techniques (Hubbard 2008). A wide body of knowledge on quantitative techniques has been accumulated over the last decades. Monte Carlo simulation is the predominant quantitative risk evaluation technique in both practice and in literature. Alternative techniques include neural networks, fuzzy logic, and decision-tree analysis. Their

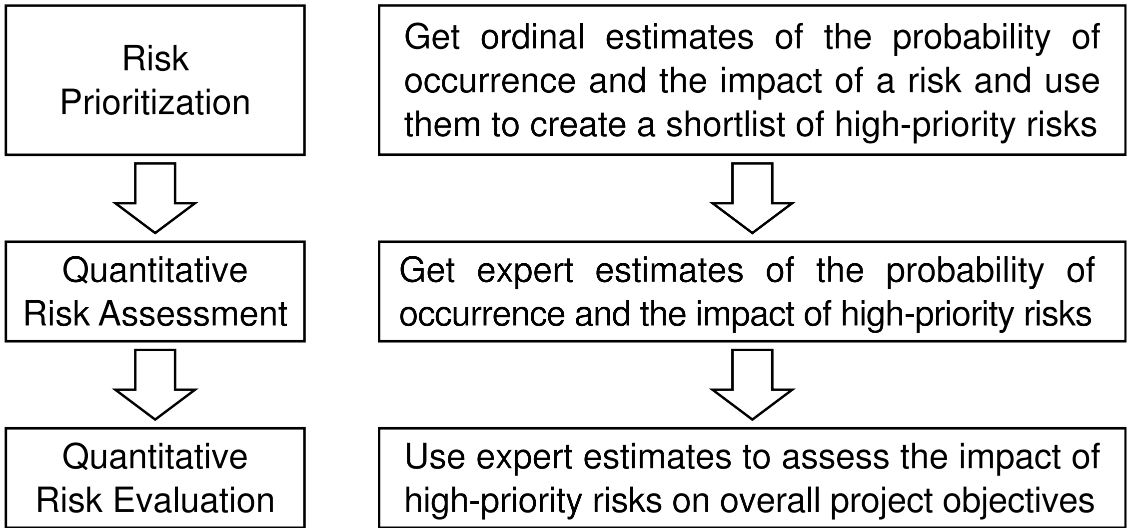

Figure 1: Risk analysis process

advocates, however, have so far failed to persuade most project schedulers of their practical use (refer to Hubbard (2008) and Chap. 61 of this handbook for an evaluation of different risk analysis techniques).

Risk analysis aims to provide insight into the risk profile of a project as to facilitate and to drive the risk response process (PMI 2008). The generated insights include: the probability of achieving a specific project outcome, the distribution of the project completion time etc. The risk response process will use these insights to come up with practical risk responses that allow project managers to mitigate risks (i.e., to reduce the impact of risks on project objectives). A popular approach to determine where to focus mitigation efforts is the use of so-called "ranking indices" (e.g., the criticality index and the significance index). Ranking indices allow the ranking of project activities (or risks) based on the impact they have on project objectives. A distinction needs to be made between activity-based ranking indices (i.e., those that rank activities) and risk-driven ranking indices (i.e., those that rank risks). Note that the ranking of the impact of an activity (or risk) may differ depending on the ranking index used. Therefore, one might wonder: which ranking index is best? In the remainder of this article, we will address this question.

This article is organized as follows: in Sect. 2 we review the basic principles of stochastic project scheduling. Sect. 3 compares the activity-based and the risk-driven approach. In Sect. 4 we present the ranking indices. Their performance is discussed in Sect. 5. Sect. 6 concludes.

2 Stochastic Project Scheduling

The Critical Path Method (CPM) has been developed in the fifties and it provides the foundations of modern project scheduling. CPM represents a project as an activity network which is a graph $G = (V, E)$ that consists of a set of nodes $V = \{1, 2, \ldots, n\}$ and a set of arcs $E = \{(i,j)|i,j \in V\}$. The nodes represent project activities whereas the arcs represent precedence relationships. Each activity i has a deterministic activity duration p_i and can only start when all its predecessors have finished. CPM uses an early-start schedule in which activities are scheduled to start as soon as possible. The early-start schedule ES is represented by a vector of earliest start times $ES = \{ES_1, ES_2, \ldots, ES_n\}$. The earliest start time of an activity i is defined as follows:

$$ES_i = \max\{EC_j|(j,i) \in E\} \tag{1}$$

Where EC_j is the earliest completion time of activity j and equals:

$$EC_j = ES_j + p_j \tag{2}$$

The project starts at time instance 0 and completes at time instance C. C is given by:

$$C = \max(EC_i|i \in V) \tag{3}$$

A path of scheduled activities is the longest path if its length equals C. A longest path is also called a critical path and the activities on the path are referred to as critical activities.

Since the fifties, many extensions of the basic model have been proposed: resource constraint project scheduling, multi-mode scheduling, generalized precedence relationships, etc. We refer to Demeulemeester and Herroelen (2002) for an extensive overview of the field. In this article we are mainly interested in what is called "stochastic project scheduling" or "stochastic CPM". Stochastic CPM acknowledges the stochastic nature of activity durations. The duration of an activity i may be represented as a random variable $\tilde{p}_i$. Because activity durations are random variables, the earliest start time and the earliest completion time are random variables as well. Let $\widetilde{ES}_i$ and $\widetilde{EC}_i$ denote the random variable of the earliest start time and the earliest completion time of an activity i respectively. The project completion time is a random variable $\widetilde{C}$ that is a function of $\tilde{p}_i$. Calculating the distribution function of $\widetilde{C}$ is shown to be $\#\mathcal{P}$-complete (Hagstrom 1988) and thus requires approximative methods such as Monte Carlo simulation (Slyke 1963). Monte Carlo simulation is used to virtually execute a project a large number of times, providing insights that can be used to enhance the actual execution of the project.

We use Monte Carlo simulation to generate random variates of $\tilde{p}_i$. More formally, let $\mathbf{p}_i = \{p_{i1}, p_{i2}, \ldots, p_{iq}\}$ denote the vector of q random variates of $\tilde{p}_i$. We refer to $\mathbf{p}_i$ as the vector of realized durations of $\tilde{p}_i$. In addition, define $\boldsymbol{ES}_i$, the vector of realized earliest start times of an activity i:

$$\boldsymbol{ES}_i = \max\{\boldsymbol{EC}_j|(j,i) \in E\} \tag{4}$$

Where $\boldsymbol{EC}_j$ is the vector of realized earliest completion times of an activity j and equals:

$$\boldsymbol{EC}_j = \boldsymbol{ES}_j + \mathbf{p}_j \tag{5}$$

The vector of realized completion times $\mathbf{C}$ is defined as follows:

$$\mathbf{C} = \max(\boldsymbol{EC}_i|i \in V) \tag{6}$$

$\boldsymbol{ES}_i$, $\boldsymbol{EC}_i$, and $\mathbf{C}$ are vectors of random variates of random variables $\widetilde{ES}_i$, $\widetilde{EC}_i$, and $\widetilde{C}$ respectively.

3 Activity-Based or Risk-Driven?

One of the biggest challenges in project risk management is to estimate and to model the uncertainty of activity durations. Often, it is assumed that the duration of an activity follows a distribution that captures all uncertainty that originates from the occurrence of risks (popular distributions include: the triangular distribution, the beta distribution and the normal distribution). As such, risk assessment boils down to providing the estimates of activity duration distribution parameters. We refer to this approach as the activity-based approach.

It has been argued that the activity-based approach is inherently flawed (Creemers et al. 2013). As pointed out by Hulett (2009), there is no clear link between the impact of identified risks on the duration of an activity and the distribution of the activity duration itself (i.e., the activity-based approach is unable to identify the root causes of the uncertainty in activity durations). In addition, it is our experience that practitioners have a hard time assessing uncertainty by estimating the parameters of an activity duration distribution.

To resolve the problems of the activity-based approach, Creemers et al. (2013) have devised a risk-driven approach in which the impact of each risk is assessed individually and is mapped to the duration of an activity afterwards. The approach is based on previous work by Schatteman et al. (2008) and Van de Vonder (2006), and is similar to the risk-driver approach of Hulett (2009). Fig. 2 illustrates the difference between the activity-based and the risk-driven approach. The activity-based approach uses Monte Carlo simulation to generate random variates of activity durations. The risk-driven approach, on the other hand, uses Monte Carlo simulation to obtain random variates of risk impacts. These random variates are then used to determine the activity durations.

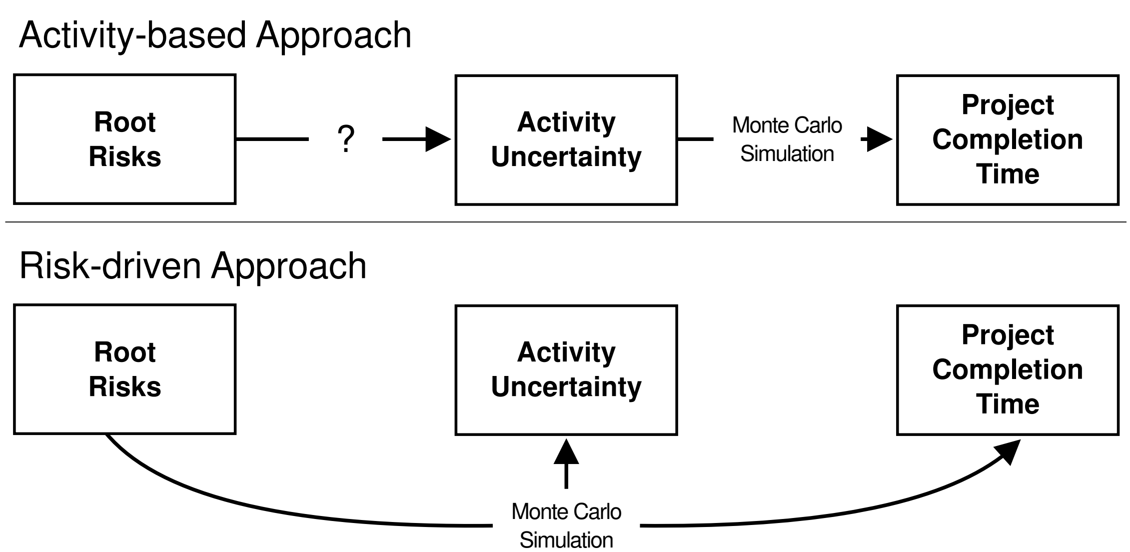

Figure 2: Activity-based versus risk-driven approach

The following example further supports the risk-driven approach. Consider an activity whose duration is impacted by two risks. The first risk has a small impact but a large probability of occurrence whereas the second risk has a large impact yet a small probability of occurrence. The probability distribution function of the duration of the activity is given in Fig. 3.

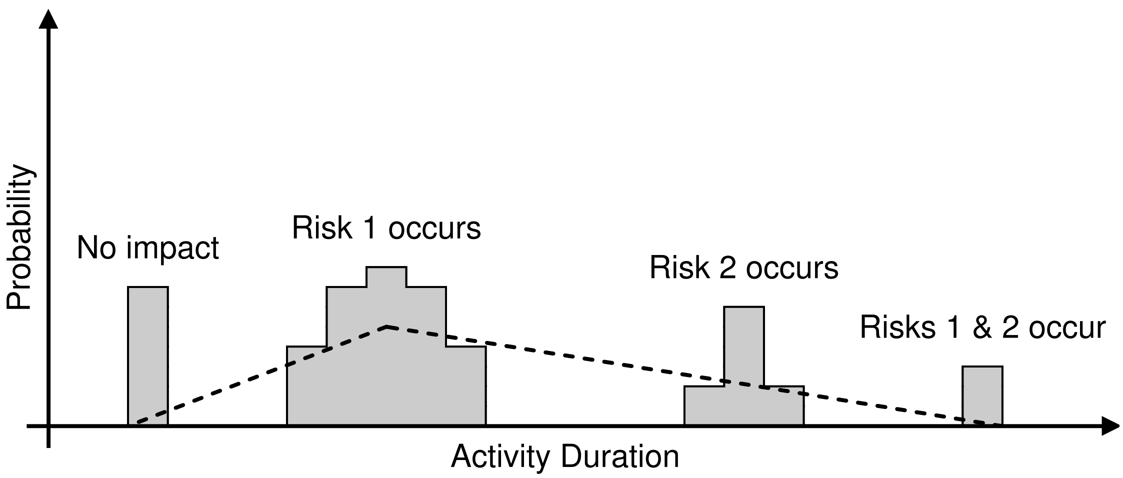

Figure 3: Example probability distribution function of the duration of an activity that is impacted by two risks

Fig. 3 also shows the best fit of the triangular distribution (i.e., the dotted line). It is clear that such a fit would result in significant errors. In addition, it would be very hard for practitioners to assess the parameters of the fitted distribution. Estimating the probability of occurrence and the impact of both risks, however, would be a manageable task.

4 Ranking Indices

Most risk analysis software packages provide the functionality to generate insight into the source of project overruns. The activities (or risks) that contribute most to the project delay are identified using ranking indices. Let $(\cdot)_i^{(W)}$ and $(\cdot)_w^{(W)}$ denote the ranking values of a ranking index $(\cdot)$ for an activity i and a risk w when activity durations are subject to a set of risks W. A large ranking value indicates that the activity (or risk) contributes a lot to the delay of the project. The ranking of activities (or risks) is typically visualized using a ranked bar chart (see Fig. 4 for an example of a ranked bar chart).

In the remainder of this section, we define how risks impact activity durations. Next, we present the ranking indices themselves. For a more detailed discussion on the ranking indices presented in this section, refer to Elmaghraby (2000), Demeulemeester and Herroelen (2002), and Creemers et al. (2013).

4.1 Definitions

In order to formally define risks and their impacts, let $R = \{1, 2, \ldots, r\}$ denote the set of risks and let $M = \left\{\widetilde{M}_{iw} | i \in V \wedge w \in R\right\}$ denote the set of risk impacts, where $\widetilde{M}_{iw}$ is

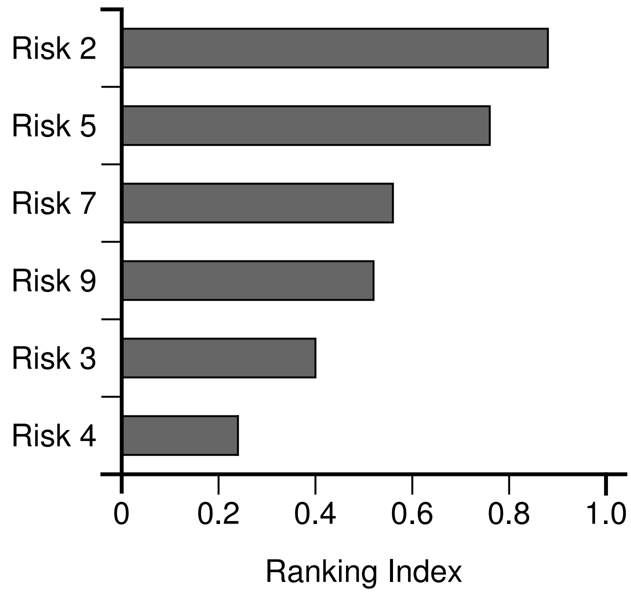

Figure 4: Example ranked bar chart

the random variable of the risk impact of a risk w on the duration of an activity i. Let $\mathbf{M}_{iw} = \{M_{iw1}, M_{iw2}, \ldots, M_{iwq}\}$ represent the vector of random variates of $\widetilde{M}_{iw}$, and define $\mathbf{p}_i^{(W)} = \left\{p_{i1}^{(W)}, p_{i2}^{(W)}, \ldots, p_{iq}^{(W)}\right\}$, the vector of random variates of the duration of an activity i when subject to a set of risks $W \subseteq R$. The entries of $\mathbf{p}_i^{(W)}$ are given by:

$$p_{ix}^{(W)} = p_i + \sum_{w \in W} M_{iwx} \qquad (x \in \{1, 2, \ldots, q\}) \tag{7}$$

Where p_i is the deterministic, risk-free duration of an activity i. From $\mathbf{p}_i^{(W)}$, we obtain $\boldsymbol{ES}_i^{(W)} = \left\{ES_{i1}^{(W)}, ES_{i2}^{(W)}, \ldots, ES_{iq}^{(W)}\right\}$, $\boldsymbol{EC}_i^{(W)} = \left\{EC_{i1}^{(W)}, EC_{i2}^{(W)}, \ldots, EC_{iq}^{(W)}\right\}$, and $\mathbf{C}^{(W)} = \left\{C_1^{(W)}, C_2^{(W)}, \ldots, C_q^{(W)}\right\}$ by generalizing Eq. 4, 5, and 6:

$$\boldsymbol{ES}_i^{(W)} = \max\left\{\boldsymbol{EC}_j^{(W)} | (j, i) \in E\right\} \tag{8}$$

$$\boldsymbol{EC}_j^{(W)} = \boldsymbol{ES}_j^{(W)} + \mathbf{p}_j^{(W)} \tag{9}$$

$$\mathbf{C}^{(W)} = \max(\boldsymbol{EC}_i^{(W)} | i \in V) \tag{10}$$

The expected project delay over q simulation iterations is defined as follows:

$$\Delta^{(W)} = \frac{1}{q} \sum_{x=1}^{q} C_x^{(W)} - C \tag{11}$$

Where C is the risk-free project completion time and is obtained using Eq. 6.

4.2 Activity-Based Ranking Indices

In what follows, we discuss the activity-based ranking indices. These indices produce a ranking of activities that may be used to determine where to focus mitigation efforts.

4.2.1 Critical Activities (CA)

It is common practice to focus mitigation efforts on the critical activities of the deterministic early-start schedule (Goldratt 1997). The Critical Activities (CA) ranking values are computed as follows:

$$CA_i^{(W)} = \delta_i \tag{12}$$

Where δ_i equals 1 if i is critical in ES and 0 otherwise.

While easy to implement, CA does not recognize the uncertain nature of a project. In addition, the discriminative power of CA is limited because all activities on the critical chain have an equal ranking value.

4.2.2 Activity Criticality Index (ACI)

In stochastic CPM, the critical path may change. The Activity Criticality Index (ACI) recognizes that almost any path and any set of activity can become critical (Slyke 1963). If Monte Carlo simulation is used, the ACI of an activity is the proportion of simulation iterations during which the activity is critical:

$$ACI_i^{(W)} = \frac{1}{q} \sum_{x=1}^{q} \delta_{ix}^{(W)} \tag{13}$$

Where $\delta_{ix}^{(W)}$ equals 1 if activity i is critical in $ES_x^{(W)}$ and 0 otherwise ($ES_x^{(W)}$ is the early-start schedule during simulation iteration x when activity durations are subject to a set of risks W).

Whereas ACI takes into account the criticality of an activity, it ignores the variance of the activity durations. Therefore, ACI cannot identify the activities that effectively contribute to the delay of the project (e.g., activities that are not impacted by risks can have a larger ACI than activities that become critical only when impacted by a risk).

4.2.3 Significance Index (SI)

The Significance Index (SI) was developed by Williams (1992) as a reaction to the criticism on ACI. If Monte Carlo simulation is used, SI is given by:

$$SI_i^{(W)} = \left(\frac{1}{\sum_{x=1}^{q} C_x^{(W)}} \right) \left[\sum_{x=1}^{q} \left(\frac{p_{ix}^{(W)}}{p_{ix}^{(W)} + TF_{ix}^{(W)}} C_x^{(W)} \right) \right] \tag{14}$$

Where $TF_{ix}^{(W)}$ is the total float of an activity i during a simulation iteration x when activity durations are subject to a set of risks W.

SI relates both the criticality of an activity and the project completion time. *SI*, however, does still not take into account the variance of the activity durations and is therefore also flawed.

4.2.4 Cruciality Index (*CRI*)

The Cruciality Index (*CRI*) is the absolute value of the correlation between the duration of an activity and the total project duration. If Monte Carlo simulation is used, *CRI* is given by:

$$CRI_i^{(W)} = \left| corr\left(\mathbf{p}_i^{(W)}, \mathbf{C}^{(W)}\right) \right| \tag{15}$$

Although rather intuitive, *CRI* has some major drawbacks. First, it measures the linear relationship between the duration of an activity and the project completion time. It is, however, well known that the relationship between these two entities does not have to be linear at all (Elmaghraby 2000). Second, *CRI* does not take into account whether or not activities are critical. As such, an activity that is not critical can have a larger *CRI* than a critical activity that has a small duration variability.

4.2.5 Spearman Rank Correlation (*SRCA*)

Cho and Yum (1997) have criticized *CRI* because of its assumption of a linear relationship between activity durations and the project completion time. They propose the use of a non-linear correlation measure such as the Spearman correlation coefficient. The Spearman Rank Correlation Index (*SRCA*) is given by:

$$SRCA_i^{(W)} = \left| corr\left(rank\left(\mathbf{p}_i^{(W)}\right), rank\left(\mathbf{C}^{(W)}\right)\right) \right| \tag{16}$$

Where $rank(\cdot)$ transforms a vector $\cdot$ into a vector of ranking numbers.

SRCA is an improvement upon *CRI* as it allows for monotonic relationships rather than linear relationships. *SRCA*, however, still does not take into account whether or not activities are critical.

4.2.6 Schedule Sensitivity Index (*SSI*)

The PMI (2008) and Vanhoucke (2010) define a ranking index that combines (1) *ACI*, (2) the variance of activity durations, and (3) the variance of the project completion time. If Monte Carlo simulation is used, the Schedule Sensitivity Index (*SSI*) is given by:

$$SSI_i^{(W)} = ACI_i^{(W)} \sqrt{\frac{var\left(\mathbf{p}_i^{(W)}\right)}{var\left(\mathbf{C}^{(W)}\right)}} \tag{17}$$

SSI captures the variance of activity durations as well as the variance of the project completion time. However, it ignores the covariance that might exist between both entities.

4.2.7 Critical Delay Contribution for Activities (*CDCA*)

Creemers et al. (2013) propose a ranking index that redistributes the project delay $\Delta^{(W)}$ over the combinations of activities and risks that cause the delay. More formally, the Critical Delay Contribution (*CDC*) of an activity i and a risk w may be expressed as follows:

$$CDC_{iw}^{(W)} = \frac{1}{q} \frac{\sum_{x=1}^{q} M_{iwx}\delta_{ix}^{(W)} \left(C_x^{(W)} - C\right)}{\sum_{i\in V}\sum_{w\in W}\sum_{x=1}^{q} M_{iwx}\delta_{ix}^{(W)}} \tag{18}$$

$$= E\left(\frac{\mathbf{M}_{iw}\mathbf{y}_i^{(W)}}{\sum_{i\in V}\sum_{w\in W}\mathbf{M}_{iw}\mathbf{y}_i^{(W)}}\right)\Delta^{(W)} \tag{19}$$

Where $\left(\mathbf{y}_i^{(W)} = \left\{\delta_{i1}^{(W)}, \delta_{i2}^{(W)}, \ldots, \delta_{iq}^{(W)}\right\}\right)$. From $CDC_{iw}^{(W)}$ it is easy to obtain an activity-based ranking index. The Critical Delay Contribution for Activities (*CDCA*) is given by:

$$CDCA_i^{(W)} = \sum_{w\in W} CDC_{iw}^{(W)} \tag{20}$$

4.3 Risk-Driven Ranking Indices

All prior ranking indices have been criticized in the literature (refer to Williams (1992), Elmaghraby (2000), Cui et al. (2006), and Creemers et al. (2013)) and are primarily designed to rank activities, not risks. To the best of our knowledge, Hulett (2009) and Creemers et al. (2013) are the only references that explicitly refer to a risk-driven ranking index. In what follows, we introduce the risk-driven ranking indices proposed by Hulett (2009) and Creemers et al. (2013).

4.3.1 Cruciality Index for Risks (*CRIR*)

Hulett (2009) proposes a simple adaptation of *CRI*. The Cruciality Index for Risks (*CRIR*) calculates the correlation between the impact of a risk and the project completion time. If Monte Carlo simulation is used, *CRIR* is given by:

$$CRIR_w^{(W)} = \left|corr\left(\mathbf{M}_w, \mathbf{C}^{(W)}\right)\right| \tag{21}$$

Where $\mathbf{M}_w = \{M_{w1}, M_{w2}, \ldots, M_{wq}\}$ and $\left(M_{wx} = \sum_{i\in V} M_{iwx}\right)$ for all $w \in W$.

4.3.2 Spearman Rank Correlation for Risks (*SRCR*)

Creemers et al. (2013) propose an adaptation of *SRCA* which is similar to the suggestion made by Hulett (2009) with respect to *CRI*. The Spearman Rank Correlation for Risks (*SRCR*) is given by:

$$SRCR_w^{(W)} = \left|corr\left(rank\left(\mathbf{M}_w\right), rank\left(\mathbf{C}^{(W)}\right)\right)\right| \tag{22}$$

4.3.3 Critical Delay Contribution for Risks (*CDCR*)

In order to compute the Critical Delay Contribution for Risks (*CDCR*), we use the *CDC*-values that were discussed in Sect. 4.2.7. *CDCR* is given by:

$$CDCR_w^{(W)} = \sum_{i \in V} CDC_{iw}^{(W)} \tag{23}$$

5 Computational Results

We perform an extensive computational experiment in order to evaluate the resilience of the ranking indices in a wide variety of settings. At the core of our experiment are the 600 projects of the PSPLIB J120 data set (Kolisch and Sprecher 1996). For each of the networks, we evaluate the mitigation potential of the ranking indices discussed in Sect. 4. A similar approach is followed in Vanhoucke (2010) and Creemers et al. (2013).

In what follow, we first discuss the experimental design and the experimental setup. Next, we define the performance measures and present the results of the computational experiment.

5.1 Experimental Design

For each of the projects of the PSPLIB J120 data set we introduce uncertainty by means of risks. We use five parameters to characterize the risks: (1) risk uniformity, (2) risk quantity, (3) risk probability, (4) risk impact, and (5) risk correlation. The settings of the parameters are based on our experience in the field of risk management.

Risk uniformity determines the number of activities that are impacted by a single risk. Often, clusters of activities have a similar task content and hence are subject to the same risks. We refer to these clusters of activities as activity groups (Schatteman et al. 2008). If risk uniformity is low, the number of activities impacted by any risk $w \in R$ follows a discrete uniform distribution with minimum and maximum equal to 1 and 3 activities respectively. A low risk uniformity setting results in an average of 60 activity groups and the average number of activities in an activity group equals 2. When risk uniformity is high, the number of activities impacted follows a discrete uniform distribution with minimum and maximum equal to 1 and 11 activities respectively. A high risk uniformity setting corresponds to an average of 20 activity groups and an average activity group size of 6 activities.

Risk quantity determines the number of risks that are identified during the risk identification process. A low risk quantity corresponds to a setting in which activities are impacted by 25 risks. When risk quantity is high, 50 risks impact the project activities. Risks are randomly assigned to a single activity group.

Risk probability determines the probability of occurrence of a risk whereas *risk impact* determines the impact of a risk on the duration of an activity. We define two types of risks: (1) risks that have a small probability of occurrence yet a large impact and (2) risks that have a large probability of occurrence but a small impact. Risks are randomly assigned to one of both risk types, where each risk has a 25 percent chance of being of type 1. Table 1 summarizes the risk probability and the risk impact settings. Note that the probability of occurrence and the risk impact are modeled using a continuous uniform distribution and a triangular distribution respectively. We opt for the use of uniform and triangular distributions because our experience learns that practitioners find it easier to assess the parameters that correspond to these distributions (e.g, a practitioner is able to assess the worst, best, and most likely impact of a risk).

Table 1: Parameter settings for risk probability and risk impact

Risk Probability	Risk Impact	Risk Type	Probability min	Probability max	Impact min	Impact most likely	Impact max
High	High	Type 1	0.05	0.05	1.0	2.0	9.0
		Type 2	0.1	0.7	0.0	1.0	2.0
High	Low	Type 1	0.05	0.05	0.5	1.0	4.5
		Type 2	0.1	0.7	0.0	0.5	1.0
Low	High	Type 1	0.025	0.025	1.0	2.0	9.0
		Type 2	0.05	0.35	0.0	1.0	2.0
Low	Low	Type 1	0.025	0.025	0.5	1.0	4.5
		Type 2	0.05	0.35	0.0	0.5	1.0

Risk correlation determines whether the occurrences of a risk (on activities in the impacted activity group) are correlated. We investigate three possible scenarios. A first scenario deals with the setting in which there is perfect correlation (i.e., either all activities in the activity group are impacted or none are). In a second scenario, we assume that the risk correlation is random, indicating that the occurrences of a risk are correlated with a random correlation factor that is drawn from a continuous uniform distribution with minimum and maximum equal to 0 and 1 respectively. The third scenario, assumes that risk occurrences are independent (i.e., there is no correlation between risk occurrences).

The settings of the five parameters combine to 48 different risk profiles that are to be evaluated. For each risk profile and over all projects in the PSPLIB J120 data set, we evaluate and compare the mitigation efficiency of the ranking indices. In this experiment, we assume that the mitigation of a risk results in the elimination of that risk.

5.2 Performance Measures

In order to compare the performance of the ranking indices, define the Relative Residual Delay (RRD) after mitigation of z risks using ranking index $(\cdot)$:

$$RRD^{(\cdot)_z} = \frac{\Delta^{(\cdot)_z}}{\Delta^{(\cdot)_0}} \tag{24}$$

Where: (1) $\Delta^{(\cdot)_z}$ is the expected project delay after mitigation of z risks using ranking index $(\cdot)$ and (2) $\Delta^{(\cdot)_0}$ is the expected project delay before any mitigation takes place. Smaller values of $RRD^{(\cdot)_z}$ correspond to a more effective ranking index.

Another measure that allows to assess the performance of a ranking index is the Mitigation Efficiency Index (MEI):

$$MEI^{(\cdot)} = 1 - 2\frac{\sum_{z=1}^{r} RRD^{(\cdot)_z}}{r-1} \tag{25}$$

The details of the dynamics of this measure may be found in Creemers et al. (2013). In short, $MEI^{(\cdot)}$ is supported on the $[-1, 1]$ real interval, where a value of 0 indicates that the performance of the ranking index equals that of a random procedure (i.e., a procedure that randomly mitigates risks). A value of 1 on the other hand, corresponds to the optimal case in which the mitigation of a single risk is sufficient to resolve all project uncertainty. In general, it is not possible to obtain a value of 1.

5.3 Experimental Setup

We test the mitigation potential of each ranking index using a stepwise procedure. In each step, the selected ranking index is used to identify the risk that contributes most to the delay of the project. Next, this risk is eliminated (i.e., is fully mitigated). After mitigation, we rerun the simulation and recalculate the expected project delay. Once more, the selected ranking index is used to identify and to mitigate the risk that has the largest impact on the project delay. This process continues until all risks have been mitigated. An outline of the procedure is provided in Algorithm 1.

We evaluate a total of 12 ranking indices. Next to the ten ranking indices discussed in Sect. 4 (i.e., CA, ACI, SI, CRI, $SRCA$, SSI, $CDCA$, $CRIR$, $SRCR$, and $CDCR$), we introduce two additional ranking indices: (1) $RAND$ randomly selects a risk from those risks still active and (2) OPT is a greedy-optimal procedure that evaluates the elimination of each risk and selects the risk that yields the largest reduction in project delay. $RAND$ may be considered as a worst-case scenario whereas OPT represents the best-case scenario. OPT, however, has limited practical value due to its computational requirements.

With respect to the activity-based ranking indices, selecting the largest risk is a two-step procedure. In a first step, the highest-ranked activity is selected. In a second step, the risk that has the largest expected impact on the selected activity is identified as the highest-ranked risk. For instance, observe the matrix of realized (during a given simulation iteration) and expected risk impacts presented in Table 2.

Table 2: Example of realized and expected risk impacts

	Realized Impacts				Expected Impacts			
Risk	1	2	3	Total	1	2	3	Total
Activity 1	2	1	1	4	1	2	1	4
Activity 2	0	0	2	2	0	0	1	1
Activity 3	0	0	1	1	0	0	1	1
Total	2	1	4	7	1	2	3	6

Algorithm 1 Computational experiment

```
for all Project networks in the PSPLIB J120 data set do
  for all Risk uniformity settings do
    Assign activities to activity groups
    for all Risk quantity settings do
      Set r and define W := {1, 2, ..., r}
      for all Risk probability settings do
        for all Risk impact settings do
          for w = 1 to r do
            Set the probability and impact of each risk
          end for
          for all Risk correlation settings do
            Set the correlation of risk occurrences
            for i = 1 to n do
              for w = 1 to r do
                for x = 1 to q do
                  For the current project and risk profile, generate common risk impact
                  M_iwx
                end for
              end for
            end for
            for all Ranking indices (·) do
              use Monte Carlo simulation to obtain Δ^(·)_z for all z : z ∈ {0, 1, ..., r}
              for all z = 0 to r do
                compute performance measure RRD^(·)_z using Eq. 24
              end for
              For the current project and the current risk profile, compute performance
              measure MEI^(·) using Eq. 25
            end for
          end for
        end for
      end for
    end for
  end for
end for
```

It is clear that activity 1 has the largest realized impact over all simulations. Risk 2 has the largest expected impact on activity 1 and hence is selected as the risk that contributes most to the project overrun (i.e., risk 2 is the highest-ranked risk). It is obvious, however, that risk 3 in fact has the most severe impact on the durations of the different activities.

In order to evaluate the performance of the different ranking indices, we use Monte Carlo simulation. The details of the simulation model are given in Creemers et al. (2013). We simulate the execution of each of the 600 projects in the PSPLIB J120 data set: (1) for each of the 48 risk profiles, (2) for each of the 12 ranking indices, and (3) for each step in the mitigation process (i.e., for each number of risks mitigated).

5.4 Results

Fig. 5 gives an overview of the average performance of the activity-based ranking indices with respect to measure $RRD^{(\cdot)_z}$ for the range starting from $z = 0$ (i.e., no risks have been mitigated) until $z = 10$ (i.e., ten risks have been mitigated). The data are aggregated over all 600 project networks in the PSPLIB J120 data set and over all 48 risk profiles. We observe that the mitigation of risks results in a decrease of the expected project delay for each ranking index. Because *RAND* randomly selects risks, its improvement is linear with the number of risks mitigated. For all other ranking indices, the improvement is convex, implying that risks with a larger impact on the project delay are selected first. One might conclude that *SRCA* is the best activity-based ranking index, closely followed by *CDCA*. It is clear, however, that there still exists a gap between the performance of the activity-based indices and the *OPT* procedure.

Fig. 6 is similar to Fig. 5 and presents the performance of risk-driven ranking indices with respect to measure *RRD*. We observe that *SRCR* outperforms *CRIR* as well as the activity-based ranking indices. More importantly, however, is the observation that *CDCR* easily outperforms *CRIR* and *SRCR*, and even matches the performance of the *OPT* procedure. It is clear that *CDCR* sets a new standard in the field of ranking indices.

Table 3 presents the *MEI* of the different ranking indices. For each ranking index, Table 3 shows: (1) the *MEI* for each risk profile and (2) the average *MEI* over the 16 risk profiles that correspond to a given risk correlation setting. We observe that the *MEI* of the *RAND* procedure is close to zero, indicating that it has no real mitigation potential. The *OPT* procedure has the highest values of *MEI* and is rivalled only by *CDCR*. Virtually no difference exists between the performance of the *OPT* procedure and the *CDCR* ranking index. With respect to the activity-based ranking indices, it is clear that *SRCA* takes the pole position, followed by *CDCA*, *ACI*, and *SI*.

Furthermore, we observe that risk correlation seems to have a limited impact on the performance of the ranking indices. A similar conclusion holds for risk probability. Risk uniformity on the other hand has a significant effect on the mitigation efficiency of the ranking indices. A higher risk uniformity results in lower performance (i.e., it is easier to distinguish between risks that only impact a small number of activities). With respect to risk quantity, we observe that an increase in the number of risks leads to a decrease of ranking index performance (i.e., if there are more risks, the mitigation of a single risk tends to be less effective). Lastly, it is clear that ranking index performance increases if risk impacts become less severe (i.e., the relative effect of mitigating a risk increases if there are only a

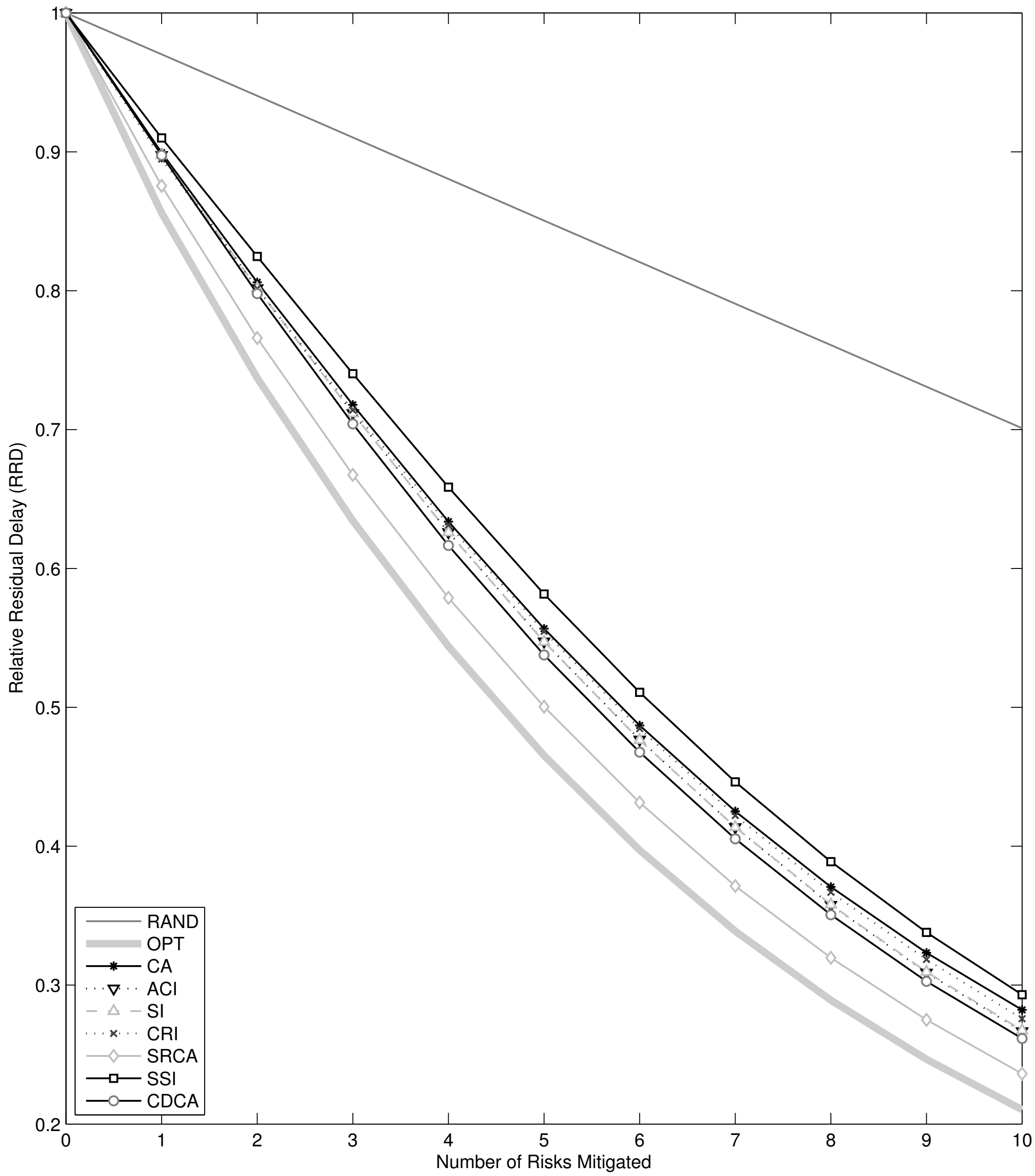

Figure 5: Mitigation potential of activity-based ranking indices

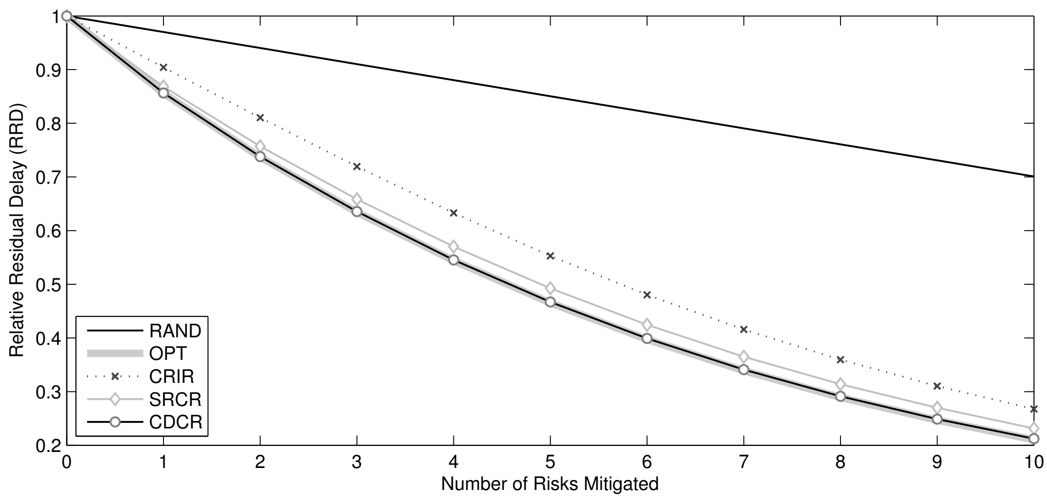

Figure 6: Mitigation potential of risk-driven ranking indices

few risks that impact project objectives).

6 Conclusions

Project risk management deals with the planning, identification, analyzing, responding, monitoring, and controlling of risks. In this article we have focussed on the risk analysis process and its effect on the risk response process. When it comes to analyzing risks, we have shown that a risk-driven approach is better than an activity-based approach. Therefore project risk management should focus on assessing the uncertainty on the level of risks (i.e., the root cause) rather than on the level of activities themselves.

In addition, we performed a large computational experiment in order to compare a number of ranking indices. Ranking indices are used to rank activities (or risks) in order to determine where to focus mitigation efforts. We compared both activity-based ranking indices (i.e., those that ranks activities) and risk-driven ranking indices (i.e., those that ranks risks). The Spearman Rank Correlation (*SRCA*) is the best activity-based ranking index. *SRCA*, however, is still far from optimal. The Critical Delay Contribution (*CDC*) may be used to devise both an activity-based and a risk-driven ranking index. The Critital Delay Contribution for Activities (*CDCA*) is second only to *SRCA* among the activity-based ranking indices. The Critital Delay Contribution for Risks (*CDCR*), on the other hand, nearly matches the performance of a greedy-optimal procedure. Therefore, we conclude that *CDCR* is the state of the art in the field of ranking indices.

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Table 3: Mitigation efficiency of the different ranking indices

Index	Avg	Corr	*MEI*															
	.621	0 %	.46	.50	.49	.52	.48	.52	.50	.53	.69	.75	.72	.76	.73	.77	.74	.78
CA	.619	100 %	.44	.49	.47	.50	.47	.51	.49	.52	.70	.75	.72	.76	.73	.77	.74	.78
	.614	RND	.45	.50	.48	.52	.48	.52	.50	.52	.70	.75	.72	.76	.73	.77	.74	.78
	.643	0 %	.49	.52	.51	.53	.51	.53	.52	.54	.74	.76	.75	.77	.77	.78	.77	.79
ACI	.640	100 %	.48	.51	.50	.52	.50	.52	.51	.53	.75	.77	.75	.77	.76	.78	.77	.79
	.637	RND	.49	.52	.50	.52	.50	.52	.51	.53	.75	.77	.76	.77	.76	.78	.77	.79
	.641	0 %	.49	.52	.51	.53	.51	.53	.52	.53	.74	.76	.75	.77	.76	.78	.77	.79
SI	.639	100 %	.48	.51	.49	.52	.50	.52	.51	.53	.74	.77	.75	.77	.76	.78	.77	.79
	.637	RND	.49	.52	.50	.52	.50	.52	.51	.53	.74	.77	.75	.77	.76	.78	.77	.79
	.612	0 %	.45	.49	.47	.50	.45	.49	.47	.50	.71	.74	.73	.75	.74	.76	.75	.77
CRI	.636	100 %	.49	.53	.52	.55	.50	.54	.53	.56	.73	.76	.75	.77	.75	.77	.76	.78
	.643	RND	.49	.53	.51	.55	.49	.52	.52	.54	.73	.75	.74	.76	.75	.77	.76	.77
	.657	0 %	.49	.53	.51	.54	.52	.55	.53	.56	.75	.78	.77	.79	.78	.81	.79	.81
SRCA	.677	100 %	.53	.57	.55	.58	.56	.59	.57	.60	.77	.79	.78	.80	.79	.81	.80	.82
	.680	RND	.53	.56	.55	.58	.55	.58	.56	.59	.76	.79	.78	.80	.79	.81	.79	.81
	.616	0 %	.46	.48	.48	.50	.46	.48	.48	.49	.73	.75	.74	.76	.75	.77	.76	.77
SSI	.614	100 %	.44	.47	.46	.49	.45	.47	.47	.48	.73	.75	.74	.76	.75	.77	.75	.77
	.610	RND	.45	.48	.48	.50	.46	.48	.47	.49	.73	.75	.74	.76	.75	.77	.76	.77
	.646	0 %	.47	.51	.49	.52	.50	.53	.51	.54	.75	.78	.76	.79	.78	.80	.79	.81
CDCA	.644	100 %	.45	.50	.47	.51	.48	.52	.50	.53	.75	.78	.76	.79	.78	.80	.78	.81
	.640	RND	.46	.51	.48	.53	.49	.53	.50	.54	.75	.78	.76	.79	.78	.80	.79	.81
	.639	0 %	.49	.53	.51	.54	.52	.55	.53	.55	.72	.75	.74	.76	.75	.77	.76	.78
CRIR	.638	100 %	.48	.51	.51	.54	.50	.53	.53	.55	.73	.75	.74	.76	.75	.77	.76	.77
	.635	RND	.49	.53	.52	.55	.50	.54	.53	.55	.73	.75	.74	.76	.75	.77	.76	.77
	.674	0 %	.52	.56	.54	.57	.56	.58	.57	.60	.75	.78	.77	.79	.78	.80	.80	.81
SRCR	.684	100 %	.50	.54	.56	.59	.55	.58	.58	.60	.75	.78	.78	.80	.79	.81	.80	.82
	.676	RND	.54	.58	.56	.60	.56	.59	.58	.60	.76	.78	.78	.80	.79	.81	.80	.82
	.697	0 %	.57	.60	.58	.61	.59	.61	.60	.62	.77	.79	.78	.80	.79	.81	.80	.82
CDCR	.695	100 %	.56	.59	.57	.60	.57	.60	.59	.61	.78	.80	.78	.80	.79	.81	.80	.82
	.692	RND	.56	.60	.58	.61	.58	.61	.59	.61	.77	.80	.78	.80	.79	.81	.80	.82
	.000	0 %	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
RAND	.000	100 %	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	.000	RND	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	.698	0 %	.57	.60	.59	.61	.59	.61	.60	.62	.77	.79	.78	.80	.79	.81	.80	.82
OPT	.697	100 %	.56	.59	.58	.60	.58	.60	.59	.61	.78	.80	.79	.80	.79	.81	.80	.82
	.695	RND	.57	.60	.58	.61	.58	.61	.59	.62	.78	.80	.79	.80	.79	.81	.80	.82
Risk Profile			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Risk Impact			H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L
Risk Probability			H		L		H		L		H		L		H		L	
Risk Quantity			H				L				H				L			
Risk Uniformity			H								L							

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