KU LEUVEN
FACULTY OF ECONOMICS AND BUSINESS

Project planning with alternative technologies in uncertain environments

Creemers S, De Reyck B, Leus R.

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Stefan Creemers
Management Department
IESEG School of Management, Lille, France
s.creemers@ieseg.fr

Bert De Reyck
Department of Management Science & Innovation
University College London, United Kingdom
bdereyck@ucl.ac.uk

Roel Leus
Research Group ORSTAT, Faculty of Economics and Business
KU Leuven, Belgium
Roel.Leus@kuleuven.be

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Abstract: We investigate project scheduling with stochastic activity durations to maximize the expected net present value. Individual activities also carry a risk of failure, which can cause the overall project to fail. To mitigate the risk that an activity's failure jeopardizes the entire project, more than one alternative may exist for reaching the project's objectives, and these alternatives can be implemented either in parallel or in sequence. In the project planning literature, such technological uncertainty is typically ignored and project plans are developed only for scenarios in which the project succeeds. We propose a model that incorporates both the risk of activity failure and the possible pursuit of alternative technologies. We find optimal solutions to the scheduling problem by means of stochastic dynamic programming. Our algorithms prescribe which alternatives need to be explored, and whether they should be investigated in parallel or in sequence. We also examine the impact of the variability of the activity durations on the project's value.

Managerial relevance: Project planning with traditional tools typically ignores technological and duration uncertainty. We explain how to model scheduling decisions in a more practical environment with considerable uncertainty, and we illustrate how decision making based only on expected values can lead to inappropriate decisions. We also refute the intuition that an increase in uncertainty necessarily entails a decrease of system performance, which seconds the proposal to focus also on 'opportunity management' rather than only on 'risk management.'

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1 Introduction

Projects in many industries are subject to considerable uncertainty, due to many possible causes. Factors influencing the completion date of a project include activities that are required but that were not identified beforehand, activities taking longer than expected, activities that need to be redone, resources being unavailable when required, late deliveries, etc. In research and development (R&D) projects, there is also the risk that activities may fail altogether, requiring the project to be halted completely. This risk is often referred to as *technical risk*. We focus on two main sources of uncertainty in R&D projects, namely uncertain activity durations and the possibility of activity failure. We examine the impact of these two factors on optimal planning strategies that maximize the project's value, and on its value itself.

We consider a single firm facing an R&D project with many possible development patterns. The project is composed of a number of R&D activities with well-defined precedence relationships, where each activity is characterized by a cost, a duration and a probability of success. The successful completion of an activity can correspond to a technological discovery or scientific breakthrough, for instance. To mitigate the risk that an activity's failure jeopardizes the entire project, the same outcome can be pursued in several different ways, where one success allows the project to continue. The different attempts can be multiple trials of the same procedure or the pursuit of different alternative ways to achieve the same outcome, e.g., the exploration of alternative technologies. These alternatives can be pursued either in parallel or sequentially, or following a mix of both strategies, and management can also decide *not* to pursue certain alternatives, for instance because their cost is too high compared to their expected benefits. Following Baldwin and Clark [5], a unit of alternative interdependent tasks with a distinguished deliverable will be called a *module*.

Project profitability is often measured by the project's net present value (NPV), the discounted value of the project's cash flows. This NPV is affected by the project schedule and therefore, the timing of expenditures and cash inflows has a major impact on the project's financial performance, especially in capital-intensive industries. The goal of this paper is to find optimal scheduling strategies that maximize the expected NPV (eNPV) of the project while taking into account the activity costs, the cash flows generated by a successful project, the variability in the activity durations, the precedence constraints, the likelihood of activity failure and the option to pursue multiple trials or technologies. Thus, this paper extends the work of Buss and Rosenblatt [11], Benati [10], Sobel et al. [45] and Creemers et al. [14], who focus on duration risk only, and of Schmidt and Grossmann [44], Jain and Grossmann [26] and De Reyck and Leus [17], who look into technical risk only (although Schmidt and Grossmann [44] also explore the possibility of introducing multiple discrete duration scenarios). Our models are also related to those of Nelson [39], Abernathy and Rosenbloom [1], Weitzman [50], Granot and Zuckerman [23], Eppinger et al. [20], Krishnan et al. [27], Dahan [16], Loch et al. [31], Ding and Eliashberg [18], Ünlüyurt [48], and Roemer and Ahmadi [43], who investigate the parallel pursuit of alternative technologies in a variety of different settings.

Our contributions are fourfold: (1) we introduce and formulate a generic model for optimally scheduling R&D activities with stochastic durations, non-zero failure probabilities and modular completion subject to precedence constraints; (2) we develop a dynamic-programming recursion to determine an optimal policy for executing the project while maximizing the project's eNPV, extending the algorithm of Creemers et al. [14] with activity failures, multiple trials and phase-type (PH) distributed activity durations instead of exponentials; (3) we conduct numerical experiments to demonstrate the computational capabilities of the algorithm; and (4) we examine the impact of activity duration risk on the optimal scheduling policy and project values. Interestingly, our findings indicate that higher operational variability does not always lead to lower project values, meaning that (sometimes costly) variance reduction strategies are not always advisable.

The remainder of this text is organized as follows. Section 2 contains an overview of related work. In Section 3, we provide the necessary definitions, leading to a detailed problem statement in Section 4. We produce solutions by means of a backward dynamic-programming recursion in a Markov decision chain, which is discussed in Section 5. Section 6 reports on our computational performance on a representative set of test instances. The effect of variability of activity durations on the eNPV of a project is subsequently informally discussed by means of an illustration in Section 7. Section 8 contains a brief summary of the paper.

2 Related work

In the absence of resource constraints, the minimization of a project's expected duration is easily accomplished by starting each activity as soon as its predecessors are completed. When the activity durations are stochastic, we are dealing with the so-called PERT problem, for which most of the literature studies the computation of certain characteristics of the minimum project makespan, mainly the exact computation, approximation and bounding of its distribution function and expected value [2, 19, 28, 33].

In a deterministic setting, project scheduling to maximize NPV has already been studied under a broad range of contractual arrangements and planning constraints (see Herroelen et al. [24] for a review). Sobel et al. [45] extend these models to incorporate stochastic activity durations and describe how to find the best policy from a finite set of scheduling policies. A similar problem is studied by Benati [10], who proposes a heuristic scheduling rule. Next to stochastic durations, Buss and Rosenblatt [11] also consider activity delays.

We incorporate the concept of *activity success or failure* into the analysis of projects with stochastic activity durations. De Reyck and Leus [17] develop an algorithm for project scheduling with uncertain activity outcomes where all activity cash flows during the development phase are negative, which is typical for R&D projects, and where project success is achieved only if all individual activities succeed. Their work constitutes the first description of an optimal approach for handling activity failures in project scheduling, but neither stochastic activity durations nor the possibility of pursuing multiple alternatives for the same result are accounted for. In that paper, if an activity A ends no later than the start of another activity B then knowledge of the outcome (success or failure) of A can sometimes be used to avoid incurring the cost for B, since a failure in A would allow abandoning the project, but payment for B cannot be avoided when B has already started before the outcome of A is discovered. For a given selection of such *information flows* between activities (under the form of additional precedence constraints), a late-start schedule is then optimal when the activity durations are known. The challenge, of course, is to find the optimal set of information flows. Earlier papers have studied optimal procedures for special cases; see Chun [12], for instance.

Unfortunately, late-start scheduling is difficult to implement in case of stochastic durations, and Sobel et al. [45] implicitly restrict their attention to scheduling policies that start activities only at the end of other activities. Buss and Rosenblatt [11] partially relax this restriction by starting an activity only after a fixed time interval (delay), but they do not decide which sets of activities to start at what time (all eligible activities are started as soon as possible after their delay). Creemers et al. [14] study the same problem as Sobel et al. [45] and achieve significant computational performance improvements. In this article, we will also restrict our attention to policies that start activities at the completion time of other activities. Finally, Coolen et al. [13] look into the properties of different classes of scheduling policies for modular networks on a single machine; since no discounting is applied, durations become irrelevant.

Other references relevant to this text stem from the discipline of chemical engineering, mainly the work by Grossmann and his colleagues (e.g., [44, 26]), who studied the scheduling of failure-prone new-product development (NPD) testing tasks when non-sequential testing is admitted. They point out that in industries such as chemicals and pharmaceuticals, the failure of a single required environmental or safety test may prevent a potential product from reaching the marketplace, which has inspired our modeling of possible activity and project failure. Therefore, our models are also of particular interest to drug-development projects, in which stringent scientific procedures have to be followed in distinct stages to ensure patient safety, before a medicine can be approved for production. Such projects may need to be terminated in any of these stages, either because the product is revealed not to have the desired properties or because of harmful side effects. Illustrations of modeling pharmaceutical projects, with a focus on resource allocation, can be found in Gittins and Yu [22] and Yu and Gittins [51].

Due to the risk of activity failure resulting in overall project failure, it has been suggested that R&D projects should explore multiple alternative ways for developing new products (Sommer and Loch [46]). Thus, we include in our model alternative technologies or multiple trials. Weitzman [50] identifies an optimal sequential scheduling strategy of alternatives, each characterized by a cost, duration, and a reward probability distribution, which maximizes the expected present value. Weitzman's *Pandora's rule* is to select the attempt with the highest reservation price (a measure that reflects each alternative's characteristics), to observe the stochastic outcome and to stop if the outcome is higher than the reservation prices of all the remaining alternatives. Granot and Zuckerman [23] examine the sequencing of R&D projects with success or failure in individual activities. Like Weitzman [50], they only consider sequential stages, but they assume that each stage is interrupted as soon one activity in that stage is successful. Nelson [39] and Abernathy and Rosenbloom [1] demonstrate the value of scheduling alternative activities in parallel. Nelson [39] aims to achieve a given objective at minimum cost. He characterizes the optimal number of parallel development activities and shows that parallelism is most beneficial when the cost of executing multiple alternatives is relatively small. Abernathy and Rosenbloom [1] look at a case with two alternatives and show that while parallel execution is more expensive, it adds value by enabling a choice between the two technologies if both are successful, and by fast-tracking the project in case one of the technologies fails.

Dahan [16] examines the trade-off between parallel and sequential scheduling of alternative prototype development methods with Bernoulli outcomes. He explores the optimal scheduling of homogeneous prototypes with identical characteristics and shows that for high discount rates the optimal policy is a hybrid policy, whereas with low discount rates a purely sequential policy is optimal. His analysis also demonstrates that extremely high or low probabilities of technical success (PTS) reduce the variability of outcomes and lead to fewer parallel prototypes being built.

Loch et al. [31] develop a dynamic-programming model to derive the optimal testing strategy for design alternatives. Taking into account the testing cost, lead time, prior knowledge and learning, they find that the optimal hybrid of parallel and sequential testing depends on the ratio of cost and time, whereby parallel processing dominates when the cost of delay increases relative to the cost of testing and development. Ding and Eliashberg [18] examine a project portfolio pipeline in which multiple projects are started simultaneously in order to increase the likelihood of having at least one successful product. In the context of concurrent engineering, parallel versus sequential scheduling of project activities has also been addressed, among others, by Eppinger et al. [20], Krishnan et al. [27] and Roemer and Ahmadi [43]. We refer to Lenfle [30] for a more extensive literature review on parallel development strategies in project management, and to Baldwin and Clark [5] for an elaborate description of the benefits of "modularity," which means splitting the design and production of technologies into independent subparts. In this text, we will take the modular structure of the project as given, assuming that an appropriate project network design has already been set out.

The idea of alternative work plans has recently also received attention in artificial intelligence: Beck and Fox [8] suggest that there may be multiple alternative process plans (routings) of an order through a factory, and they investigate how to incorporate this issue into scheduling decisions. Bartak and Cepek [6] and Bartak et al. [7] continue this work. In the literature on production management and automation, similar ideas of alternative routings have been studied; see, for instance, Ahn et al. [3], Pan and Chen [42], Nasr and Elsayed [38] and Kusiak and Finke [29]. To the best of our knowledge, however, the literature on alternative process plans does not consider duration uncertainty nor the possibility of activity failures.

The way in which we model project success in this paper is similar to the literature on reliability systems, especially to the sequential testing of components of a multi-component system in order to learn the state of the system when the tests are costly; a comprehensive review of this area is provided in Ünlüyurt [48]. In these works, all activities (tests) are performed in series and discounting is not considered; these two characteristics actually go hand in hand, since it is a dominant decision to execute all activities in series when money has no time value (see De Reyck and Leus [17]). In the operational scheduling literature, different approaches to scheduling precedence-related tasks with re-attempt at failure can be found in Mori and Tseng [37] and Malewicz [34, 35].

3 Definitions

A project consists of a set of activities $N = \{0, \ldots, n\}$. Some of these activities are alternatives that pursue a similar target, representing repeated trials or technological alternatives; such activities are gathered in one module. The set of modules is $M = \{0, \ldots, m\}$; each module $i \in M$ contains the activities $N_i \subset N$, and the set of modules constitutes a partition of N: $N = \bigcup_{i \in M} N_i$ and $N_i \cap N_j = \varnothing$ if $i \neq j$. Activities should be executed without interruption. A is a (strict) partial order on M, i.e., an irreflexive and transitive relation, which represents technological precedence constraints. (Dummy)

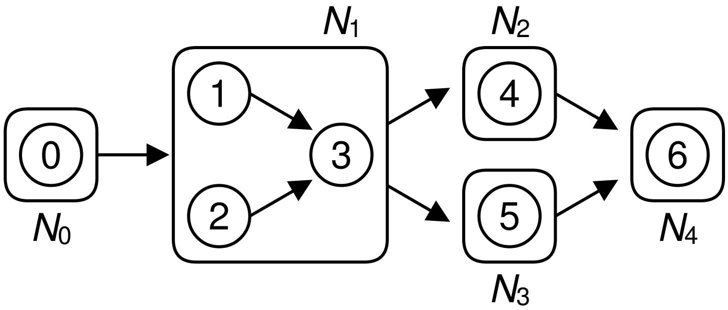


Figure 1: Example module network

modules 0 and m represent the start and the end of the project, respectively; they are the (unique) least and greatest element of the partially ordered set (M, A) and are assumed to contain only one (dummy) activity, indexed by 0 and n, respectively. On the activities within each module i, we also impose a partial order B_i, to allow for modeling precedence requirements between these activities. Figure 1 illustrates these definitions. The project consists of seven activities, $N = \{0, 1, 2, 3, 4, 5, 6\}$, where 0 and $n = 6$ are dummies. There are five modules, so $m = 4 : N_0 = \{0\}, N_1 = \{1, 2, 3\}, N_2 = \{4\}$, $N_3 = \{5\}$ and $N_4 = \{6\}$. In the example, $B_1 = \{(1, 3), (2, 3)\}$. Note that Figure 1 actually shows the transitive reduction of A: the order relation A also contains elements such as $(0, 2)$ and $(1, 4)$, while the arcs $N_0 \to N_2$ and $N_1 \to N_4$ are not included in the figure.

Each activity $i \in N \backslash N_m$ has a PTS p_i; we assume that $p_0 = 1$. We do not consider (renewable or other) resource constraints and assume the outcomes of the different tasks to be independent. The cash flow associated with the execution of activity $i \in N \backslash N_m$ is represented by the integer value c_i and is incurred at the start of the activity. These c_i are usually non-positive, although the algorithm developed in Section 5 is able to handle both positive and negative intermediate cash flows simultaneously. We choose $c_0 = 0$. If the project is successful, i.e., if every module is successful, it generates an end-of-project payoff $C \geq 0$, which is received at the start of activity n. A module is successful if at least one of its constituent activities succeeds. Without loss of generality, we assume that C is large enough for the project to be undertaken (otherwise, all starting times may be set to infinity).

We define a *success (state) vector* as an n-component binary vector $\mathbf{x} = (x_0, x_1, \ldots, x_{n-1})$, with one component associated with each activity in $N \setminus \{n\}$. We let X_i represent the Bernoulli random variable with parameter p_i as success probability for each activity i, and we write $\mathbf{X} = (X_0, X_1, \ldots, X_{n-1})$. Information on an activity's success (the realization of X_i) becomes available only at the end of that activity. We say that $\mathbf{x}$ is a *realization* or *scenario* of $\mathbf{X}$ (the literature sometimes also uses the term *random variate*). The duration $D_j \geq 0$ of each activity j is also a stochastic variable; the vector $(D_0, D_1, \ldots, D_{n-1})$ is denoted by $\mathbf{D}$. We use lowercase vector $\mathbf{d} = (d_0, d_1, \ldots, d_{n-1})$ to represent one particular realization of $\mathbf{D}$, and we assume $Pr[D_0 = 0] = 1$. For the remaining activities $j \in N \setminus \{0, n\}$, the durations D_j are mutually independent PH-distributed stochastic variables (see Section 5.3 for more details).

The value $s_i \geq 0$ represents the starting time of activity i; we call the $(n+1)$-vector $\mathbf{s} = (s_0, \ldots, s_{n-1}, s_n)$ a schedule,with $s_i \geq 0$ for all $i \in N$. We assume $s_0 = 0$ in what follows: the project starts at time zero. For convenience, we associate a completion time $e_i(\mathbf{s}; \mathbf{d}, \mathbf{x})$ with each module i, in the following way (here and later, we omit the arguments if no misinterpretation is possible): $e_i = \min\{+\infty; \min_{j \in N_i | x_j = 1}\{s_j + d_j\}\}$. Clearly, if the second min-operator optimizes over the empty set then $e_i = +\infty$, implying that the module is never successfully completed. For a given success vector $\mathbf{x}$ and durations $\mathbf{d}$, we say that a schedule $\mathbf{s}$ is *feasible* if the following conditions are fulfilled:

$$e_i \leq s_j \qquad \forall (i,k) \in A, \forall j \in N_k \tag{1}$$

$$s_i + d_i \leq s_j \qquad \forall k \in M, \forall (i,j) \in B_k \tag{2}$$

Equations (1) are inter-module precedence constraints, while Equations (2) are intra-module constraints. An activity's starting time equal to infinity corresponds to not executing the activity and therefore not incurring any related expenses, or in case of activity n, not receiving the project payoff. In words, the necessary conditions for the start of an activity $i \in N_j$ are (1) success for all the predecessor modules of the module j to which i belongs, and (2) completion of all predecessor activities in the same module.

We compute the NPV for schedule $\mathbf{s}$ as

$$f(\mathbf{s}) = Ce^{-rs_n} + \sum_{i=1}^{n-1} c_i e^{-rs_i}, \tag{3}$$

with r a continuous discount rate chosen to represent the time value of money: the present value of a cash flow c incurred at time t equals ce^{-rt}. Here and below, we assume $e^{-0 \cdot \infty} = 0$. Note that symbol e is the base of the natural logarithm here, while above it was used for module completion times; there will be little danger of confusion.

4 Problem statement

The execution of a project with stochastic components (in our case, stochastic activity outcomes and durations) is a dynamic decision process. A solution, therefore, cannot be represented by a schedule but takes the form of a *policy*: a set of decision rules defining *actions* at *decision times*, which may depend on the prior outcomes. Decision times are typically the start of the project and the completion times of activities; a tentative next decision time can also be specified by the decision maker. An action entails the start of a precedence-feasible set of activities (no constraints in (1) or (2) are violated). In this way, a schedule is constructed gradually as time progresses. The following requirement, the so-called *non-anticipativity constraint*, is also imposed: next to the information available at the start of the project, a decision at time t can only use information on duration realizations and realizations of components of $\mathbf{X}$ that has become available before or at time t. The technical concept of a 'policy' corresponds to the practical notion of a project *strategy*, which can be defined as a 'direction' in a project (goal, plan, guideline, ...) that contributes to success of the project in its environment (Artto et al. [4]).

We follow Igelmund and Radermacher [25], Möhring [36] and Stork [47], who study project scheduling with resource constraints and stochastic activity durations, in interpreting every policy Π as a function $\mathbb{R}^n_{\geq} \times \mathbb{B}^n \to \mathbb{R}^{n+1}_{\geq}$, with $\mathbb{R}_{\geq}$ the set of non-negative reals and $\mathbb{B} = \{0, 1\}$. The function Π maps given samples $(\mathbf{d}, \mathbf{x})$ of activity durations and success vectors to vectors $\mathbf{s}(\mathbf{d}, \mathbf{x}; \Pi)$ of feasible activity starting times (schedules). For a given duration scenario $\mathbf{d}$, success vector $\mathbf{x}$ and policy Π, $s_n(\mathbf{d}, \mathbf{x}; \Pi)$ denotes the makespan of the schedule, which coincides with project completion. Note that not all activities need to be completed (or even started) by s_n, nor that the realization of all X_i needs to be known. The earlier-mentioned PERT problem aims at characterizing the random variable $s_n(\mathbf{D}, \mathbf{1}; \Pi^{ES})$, where policy Π^{ES} starts all activities as early as possible, each module contains only one activity, and $\mathbf{1}$ is an n-vector with value 1 in all components. Contrary to the makespan, however, NPV is a non-regular measure of performance: starting activities as early as possible is not necessarily optimal, since the c_i may be negative.

Our goal is to select a policy Π^* that maximizes $\mathbb{E}[f(\mathbf{s}(\mathbf{D}, \mathbf{X}; \Pi))]$, with $f(\cdot)$ the NPV-function as defined in Equation (3) and $\mathbb{E}[\cdot]$ the expectation operator with respect to $\mathbf{D}$ and $\mathbf{X}$; we write $\mathbb{E}[f(\Pi)]$, for short. The generality of this problem statement suggests that optimization over the class of all policies is probably computationally intractable. We therefore restrict our optimization to a subclass that has a simple combinatorial representation and where decision points are limited in number: our solution space $\mathcal{P}$

consists of all policies that start activities only at the end of other activities (activity 0 is started at time 0).

For the example that was introduced in Section 3, a possible policy Π_0 is the following: start the project at time 0 ($s_0 = 0$) and immediately initiate activities 1 and 2 ($s_1 = s_2 = 0$). If $X_1 = X_2 = 0$ then abandon the project: set $s_3 = s_4 = s_5 = s_6 = +\infty$. Otherwise, module N_1 completes successfully. In that case, start both activities 4 and 5 upon the successful completion of activity 1 or 2 (whichever is the earliest), and terminate the project if either 4 or 5 fails. Note that under policy Π_0, activity 3 is never started, and we effectively include activity *selection* as part of the decisions to be made. Represented as a function, Π_0 entails the following mapping:

$$\begin{gathered}(d_0, d_1, d_2, d_3, d_4, d_5, x_0, x_1, x_2, x_3, x_4, x_5)\\ \mapsto\\ (0, 0, 0, \infty, e_1, e_1, \max\{e_2; e_3\}),\end{gathered}$$

with $e_1 = \min\{d_1 + \infty \cdot (1 - x_1); d_2 + \infty \cdot (1 - x_2)\}$, $e_2 = e_1 + d_4 + \infty \cdot (1 - x_4)$ and $e_3 = e_1 + d_5 + \infty \cdot (1 - x_5)$. In the context of this article, we let $0 \cdot \infty = 0$.

5 Markov decision chain

In the literature, the input parameters of the PERT problem are often referred as a PERT network, and a PERT network with independent and exponentially distributed activity durations is also called a *Markovian PERT network*. For Markovian PERT networks, Kulkarni and Adlakha [28] describe an exact method for deriving the distribution and moments of the earliest project completion time using continuous-time Markov chains (CTMCs). Buss and Rosenblatt [11], Sobel et al. [45] and Creemers et al. [14] investigate an eNPV objective and use the CTMC described by Kulkarni and Adklakha as a starting point for their algorithms. These studies, however, assume success in all activities and an exponential distribution for all durations and they also imply the requirement that all activities be executed. In this article, we will extend the work of Creemers et al. [14] to accommodate PH-distributed activity durations, possible activity failures and a modular project network, allowing also for activity selection. Below, we first study the special case of exponential activity durations (Section 5.1), followed by an illustration (Section 5.2) and by a treatment of more general distributions (Section 5.3).

5.1 The exponential case

For the moment, we assume each duration D_i to be exponentially distributed with rate parameter $\lambda_i = 1/\mathbb{E}[D_i]$ $(i = 1, \ldots, n-1)$; we consider more general distributions in Section 5.3. At any time instant t, an activity's status is either *idle* (not yet started), *active* (being executed), or *past* (successfully finished, failed, or considered redundant because its module is completed). Let $I(t)$, $Y(t)$ and $P(t)$ represent the activities in N that are idle, active and past, respectively; these three sets are mutually exclusive and $I(t) \cup Y(t) \cup P(t) = N$. The state of the system is defined by the status of the individual activities and is represented by a triplet (I, Y, P). State transitions take place each time an activity becomes past and are determined by the policy at hand. The project's starting conditions are $Y(0) = \{0\}$ and $I(0) = N \setminus \{0\}$, while the condition for *successful* completion of the project is $P(t^*) = N$, where t^* represents the project completion time.

The problem of finding an optimal scheduling policy corresponds to optimizing a discounted criterion in a continuous-time Markov decision chain (CTMDC) on the state space Q, with Q containing all the states of the system that can be visited by the transitions (which are called *feasible* states); the decision set is described below. We apply a backward stochastic dynamic-programming (SDP) recursion to determine optimal decisions based on the CTMC described in Kulkarni and Adlakha [28]. The key instrument of the SDP recursion is the *value function* $F(\cdot)$, which determines the expected NPV of each feasible state at the time of entry of the state, conditional on the hypothesis that optimal decisions are made in all subsequent states and assuming that all 'past' modules (with all activities past) were successful. In the definition of the value function $F(I, Y)$, we supply sets I and Y of idle and active activities as parameters (which uniquely determines the past activities). When an activity finishes, three different state transitions can occur: (1) activity $j \in N_i$ completes successfully; (2) activity $j \in N_i$ fails and another activity $k \in N_i$ is still idle or active; (3) activity $j \in N_i$ fails and all other activities $k \in N_i$ have already failed (or it is the only activity in the module).

We define the order B^* on set N to relate activities that do not necessarily belong to the same module, as follows:

$$(i, j) \in B^* \Leftrightarrow (\exists B_m : (i, j) \in B_m) \vee (\exists (l, m) \in A : i \in N_l \wedge j \in N_m).$$

We call an activity j *eligible* at time t if $j \in I(t)$ and $\forall (k, j) \in B^* : k \in P(t)$. Let $E(I, Y) \subset N$ be the set of eligible activities for given sets I and Y of idle and active activities. Upon entry of a state $(I, Y, P) \in Q$, a decision needs to be made whether or not to start eligible activities in $E(I, Y)$ and if so, which. If no activities are started, a transition towards another state occurs at the first completion of an element of Y. Not starting any activities while there are no active activities left, corresponds to abandoning the project. Let $\hat{\lambda} = \sum_{k \in Y} \lambda_k$. The probability that activity $j \in Y$ completes first among the active activities equals $\lambda_j / \hat{\lambda}$ (see Appendix 1). The expected time to the first completion is $\hat{\lambda}^{-1}$ time units (the length of this timespan is also exponentially distributed) and the appropriate discount factor to be applied for this timespan is $\hat{\lambda} / \left(r + \hat{\lambda}\right)$ (cfr. appendix). In state $(I, Y, P) \in Q$, the expected NPV to be obtained from the next state on condition that no new activities are started equals

$$
\begin{aligned}
&\frac{\hat{\lambda}}{r + \hat{\lambda}} \sum_{j \in Y} \frac{p_j \lambda_j}{\hat{\lambda}} F(I \setminus N_i, Y \setminus N_i) + \\
&\frac{\hat{\lambda}}{r + \hat{\lambda}} \sum_{j \in Y : N_i \setminus \{j\} \not\subset P} \frac{(1 - p_j)\lambda_j}{\hat{\lambda}} F(I, Y \setminus \{j\}),
\end{aligned} \tag{4}
$$

with $j \in N_i$ in the summations. Our side conditions are $F(I, \varnothing) = 0$ for all I.

The second alternative is to start a non-empty set of eligible activities $S \subseteq E(I, Y)$ when a state $(I, Y, P) \in Q$ is entered. This leads to incurring a cost $\sum_{j \in S} c_j$ and an immediate transition to another state, with no discounting required. The corresponding eNPV, conditional on set $S \neq \varnothing$ being started, is

$$
F(I \setminus S, Y \cup S) + \sum_{j \in S} c_j. \tag{5}
$$

The total number of decisions S that can be made is $2^{|E(I,Y)|}$. The decision corresponding to the highest value in (4) and (5) determines $F(I, Y)$.

The backward SDP recursion starts in state $(\varnothing, \{n\}, N \setminus \{n\})$. Subsequently, the value function is computed stepwise for all other states. The optimal objective value $\max_{\Pi \in \mathcal{P}} \mathbb{E}[f(\Pi)]$ is obtained as $F(N \setminus \{0\}, \{0\})$. We should note that the policies from which one with the best objective function is chosen, do not consider the option of starting activities at the end of activities that are redundant (past) because another activity already made their module succeed.

Table 1: Project data for the example project

task i	cash flow c_i	mean duration $\mathbb{E}[D_i]$	PTS p_i
0	0	0	100%
1	-20	10	40%
2	-35	2	35%
3	-70	8	75%
4	-10	2	100%
5	-10	2	60%
6		0	100%

5.2 Illustration

In this section, we illustrate the functioning of the SDP algorithm by analyzing the example project with seven activities ($n = 6$) introduced in Section 3, for which the module order A is described by Figure 1. Further input data are provided in Table 1; the project's payoff value C is 300 and the discount rate is 10 percent per time unit ($r = 0.1$).

For exponentially distributed activity durations, the SDP recursion described in Section 5.1 can be applied to find an optimal policy. At the onset of the project (in state $(N \setminus \{0\}, \varnothing, \{0\})$) we can decide to start either the first activity, the second activity, or both, from module 1. The SDP recursion evaluates the expected outcome of each of these decisions and selects one that yields the highest expected NPV (assuming that optimal decisions are made at all future decision times). In our example, it is optimal to start only the first activity (corresponding to an objective function of 3.27) and we subsequently end up in state $(\{2,3,4,5\}, \{1\}, \{0\})$, in which two possibilities arise. If activity 1 succeeds, module 1 succeeds as well and a transition occurs to state $(\{4,5\}, \varnothing, \{0,1,2,3\})$; otherwise (if activity 1 fails), we end up in state $(\{2,3,4,5\}, \varnothing, \{0,1\})$ and have to make a decision: either we start activity 2, corresponding to a transition to state $(\{3,4,5\}, \{2\}, \{0,1\})$ and an eNPV at that time for the remainder of the project of -1.06, or we abandon the project altogether obtaining a current value of 0. The optimal decision in this case is obviously not to continue the project.

After a successful completion of module 1, two new activities become eligible. The optimal decision is to start both activities 4 and 5, leading to state $(\varnothing, \{4,5\}, \{0,1,2,3\})$. Two possibilities then arise: either activity 4 or activity 5 finishes first. Irrespective of which activity completes first, if either activity 4 or 5 fails then the entire project fails. If activity 4 (resp. 5) finishes first and succeeds, activity 5 (resp. 4) is still in progress and needs to finish successfully for the project payoff to be earned. We refer to this optimal policy for exponential durations by the name Π_1.

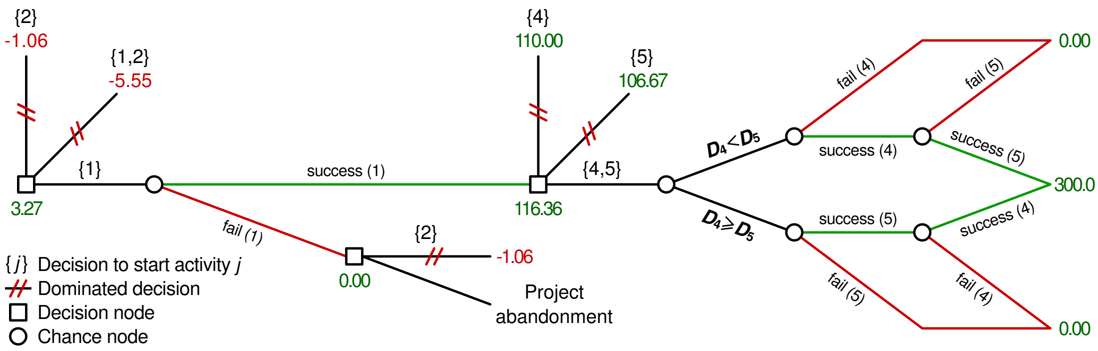


Figure 2: Optimal paths in the decision tree for the example project

The relevant part of the corresponding decision tree is represented in Figure 2, in which the project evolves from left to right. A *decision node*, represented by a square, indicates that a decision needs to be made at that point in the process; a *chance node*, denoted by a circle, indicates that a random event takes place. Underneath each decision node, we indicate the eNPV conditional on an optimal decision being made in the node, which applies only to the part of the project that remains to be performed. For each decision node, a double dash // is added to each branch that does not correspond to an optimal choice in the SDP recursion.

5.3 Generalization towards PH distributions

PH distributions were first introduced by Neuts [40] as a means to approximate general distributions using a combination of exponentials. We will adopt so-called acyclic PH distributions for the activity durations in order to assess the impact of activity duration variability on the eNPV of a project. In this section, we informally describe PH distributions and show how to determine the optimal eNPV of a project when activity durations are PH distributed. Formal definitions and a moment-matching approach that can used to approximate an arbitrary distribution are included as Appendix 2 and Appendix 3.

Due to the properties of the acyclic PH distribution, each activity $j \neq 0, n$ can be seen as a sequence of z_j phases where:

- each phase θ_{ju} has an exponential duration with rate λ_{ju},
- each phase θ_{ju} has a probability τ_{ju} to be the initial phase when starting activity j,
- each phase θ_{ju} is visited with a given probability π_{jvu} when departing from another phase θ_{jv}.

Acyclicity of the distribution implies that a state is never visited more than once. Since the execution of a task is non-preemptive, the execution of the sequence of phases as well as the execution of a phase itself should be uninterrupted. Therefore, upon completion of a phase θ_{ju}:

- activity j completes with probability π_{ju0} (absorption is reached in the underlying Markov chain),
- phase v is started with probability π_{juv}.

The exponential distribution for activity $j \in N \setminus \{0, n\}$ is then a PH distribution with $z_j = 1$, $\tau_{j1} = 1$ and $\lambda_{j1} \equiv \lambda_j$.

Maintaining the definition of $Y(t)$ given in Section 5.1, define $Y^\circ(t)$ as the set of phases of the activities in $Y(t)$ that are being executed at time instant t. Clearly, Y can be obtained from Y°. The state of the system is again fully determined by the status of the individual activities and is now represented by a triplet (I, Y°, P). The SDP recursion described in the previous subsection for computing function F is easily extended to accommodate PH distributions; the most important modification is in Equation (4), which becomes

$$\begin{aligned}
&\frac{\hat{\lambda}^\circ}{r+\hat{\lambda}^\circ} \sum_{\theta_{ju}\in Y^\circ} \pi_{ju0}\frac{p_j\lambda_{ju}}{\hat{\lambda}^\circ} F(I\setminus N_i, Y^\circ \setminus N_i^\circ)+ \\
&\frac{\hat{\lambda}^\circ}{r+\hat{\lambda}^\circ} \sum_{\theta_{ju}\in Y^\circ : N_i\setminus\{j\}\not\subset P} \pi_{ju0}\frac{(1-p_j)\lambda_{ju}}{\hat{\lambda}^\circ} F(I, Y^\circ \setminus \{\theta_{ju}\})+ \\
&\frac{\hat{\lambda}^\circ}{r+\hat{\lambda}^\circ} \sum_{\theta_{ju}\in Y^\circ} \frac{\lambda_{ju}}{\hat{\lambda}^\circ} \sum_{\substack{v=1\\ v\neq u}}^{z_j} \pi_{juv} F(I, Y^\circ \cup \{\theta_{jv}\} \setminus \{\theta_{ju}\}),
\end{aligned} \tag{6}$$

with $j \in N_i$, $\hat{\lambda}^\circ = \sum_{\theta_{kv}\in Y^\circ} \lambda_{kv}$ and $N_i^\circ = \{\theta_{ku} : k \in N_i\}$. We use the result that the probability that phase $\theta_{ju} \in Y^\circ$ completes first among the active phases equals $\lambda_{ju}/\hat{\lambda}^\circ$ and that the expected time to the first completion is ${\hat{\lambda}^\circ}^{-1}$ time units.

6 Computational performance

In this section, we will briefly evaluate the computational performance of the SDP algorithm. For exponential durations, an upper bound on $|Q|$ is 3^n. Enumerating all these 3^n states is not recommendable, because typically the majority of the states do not satisfy the precedence constraints. For PH durations, the bound becomes $\prod_{j\in N} 3^{z_j}$. In order to minimize storage and computational requirements, we adopt the techniques proposed by Creemers et al. [14]: as the algorithm progresses, the information on the earlier generated states will no longer be required for further computation and therefore the memory occupied can be freed. This procedure is based on a partition of Q, allowing for the necessary subsets to be generated and deleted when appropriate.

We borrow the datasets that were generated by Coolen et al. [13]: these consist of 10 instances for each of various values of the number of activities n and for $OS = 0.4$, 0.6 and 0.8, with 'order strength' OS the number of comparable activity pairs according to the induced order B^*, divided by the maximum possible number of such pairs (this value is only approximate). Average activity durations are not used by Coolen et al. [13] and are additionally generated for each activity, for each instance separately; each such average duration is a uniform integer random variate between 1 and 15. The sign of the cash flows is unimportant to our algorithm; in the generated instances, all activities apart from the final one have negative cash flows.

In our implementation, storage requirement for $600,000$ states amounts to a maximum of 4.58 MB; we only generate feasible states. Our experiments are performed on an AMD Phenom II with 3.21 GHz CPU speed and $2,048$ MB of RAM. Under this configuration, a state space of maximum $268,435,456$ states can be entirely stored in memory. Our results are presented in Tables 2-4, gathered per combination of values for OS and n. The tables show that networks of up to 40 activities are analyzed with relative ease. When $n = 51$, however, the optimal solution of most networks with low order strength ($OS = 0.4$) is beyond reach when the system memory is restricted to $2,048$ MB. When $OS = 0.6$, the performance is limited to networks with $n = 71$ or less. We observe that the density of the induced order B^* is a major determinant for the computational effort: order strengths and computation times clearly display an inverse relation.

7 Impact of activity duration variability

In this section, we examine the impact of different degrees of variability of the activity durations for the example project instance. The policy Π_1 described in Section 5.2 is optimal with exponential durations; its objective value is 3.27. The quality of the policy changes when the variability level is different, however. Figure 3(a) illustrates the functioning of policy Π_1 with deterministic durations: the policy first executes only activity 1, and then starts both activity 4 and 5 if 1 succeeds, otherwise the project is abandoned.

Table 2: Number of successfully analyzed networks out of 10

n	$OS = 0.8$	$OS = 0.6$	$OS = 0.4$
11	10	10	10
21	10	10	10
31	10	10	10
41	10	10	7
51	10	10	5
61	10	6	3
71	9	5	3
81	10	4	1
91	9	4	0
101	10	1	0
111	9	1	0
121	8	0	0

Table 3: average size of the state space ($|Q|$) for analyzed networks

n	$OS = 0.8$	$OS = 0.6$	$OS = 0.4$
11	74	248	628
21	396	4,303	29,793
31	2,174	192,984	911,558
41	15,871	1,619,351	25,051,988
51	98,559	1,940,598	90,057,422
61	177,916	29,540,126	278,145,443
71	2,260,271	85,611,285	82,971,948
81	2,070,967	34,261,271	176,976,352
91	23,128,416	145,911,293	
101	24,804,064	165,306,852	
111	67,477,195	56,193,712	
121	69,245,416		

The objective function value for Π_1 with deterministic durations ($\mathbb{E}[f(\Pi_1)]$) is

$$c_1 + p_1 e^{-r\mathbb{E}[D_1]} \left(c_4 + c_5 + p_4 p_5 C e^{-r\mathbb{E}[D_5]} \right) = -1.26.$$

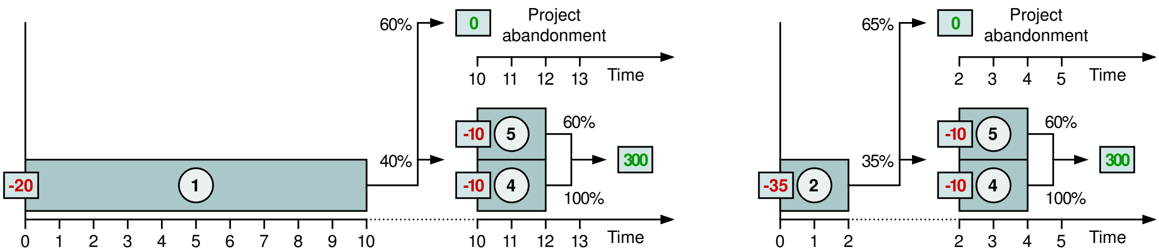
An optimal policy Π_2 for this setting is described by Figure 3(b), with eNPV ($\mathbb{E}[f(\Pi_2)]$)

$$c_2 + p_2 e^{-r\mathbb{E}[D_2]} \left(c_4 + c_5 + p_4 p_5 C e^{-r\mathbb{E}[D_5]} \right) = 1.50.$$

Here, activity 2 is started at the project's initiation, and activity 1 is never selected (i.e., upon failure of activity 2 the project is abandoned). With exponential durations, on the other hand, Π_2 has an objective value of -1.06. Interestingly, the inferior policy in the case of exponential durations becomes optimal when activity durations are deterministic.

Table 4: average CPU time required to find an optimal policy

n	$OS = 0.8$	$OS = 0.6$	$OS = 0.4$
11	0	0	0
21	0	0	0.03
31	0	0.3	1.77
41	0.02	3.54	70.93
51	0.15	5.12	298.41
61	0.32	128.31	2,397.93
71	17.53	469.34	27,065.53
81	5.7	1817.54	15,605.91
91	107.61	1,322.77	
101	105.66	894.61	
111	283.57	10,540.86	
121	528.81		

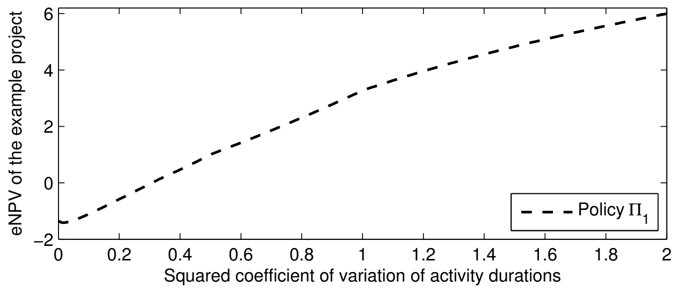


(a) Policy Π_1 (b) Policy Π_2

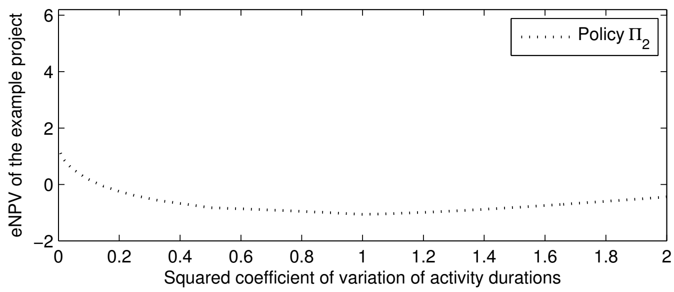
Figure 3: Policies with deterministic durations

Also, the effect of variability on the eNPV associated with a policy is not monotonic; the eNPV of policy 1 increases, whereas the eNPV of policy 2 decreases. Of particular interest is the fact that the eNPV can actually increase when variability is introduced, which is quite counterintuitive. Note also that for each of the two variability settings, the sign of the objective of two policies is different (one policy achieves a negative NPV while the other one has positive NPV). This is a strong case for incorporating all variability information into the computations and not only 'plugging in' the expectations into a deterministic model, since a good project might be cut from the portfolio based only expected values, whereas it would be able to add value with a carefully selected scheduling strategy.

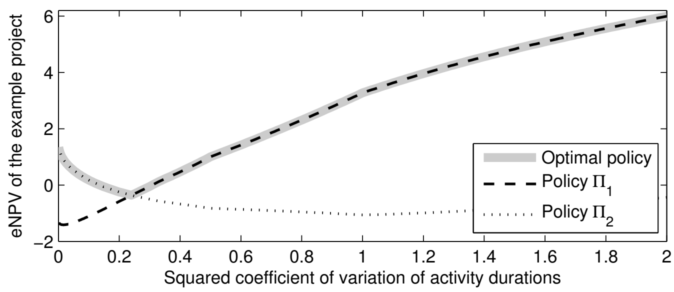
To investigate the impact of variability in more detail, we use PH distributions to model the activity durations, which will allow us to increase or decrease the variability and examine its impact on the project's eNPV, by changing the Squared Coefficient of Variation (SCV) of the activity durations (for simplicity, we assume all activity durations to have equal SCV). Setting SCV equal to 1 corresponds to exponentially distributed



(a) effect on eNPV of policy Π_1



(b) effect on eNPV of policy Π_2



(c) effect on optimal eNPV

Figure 4: The effect of activity duration variability for the example project

activity durations, an SCV equal to 0 corresponds to deterministic durations.

Figures 4(a) and 4(b) report the eNPV of the example project for various values of SCV, executed according to policy Π_1 and policy Π_2, respectively. We observe that eNPV decreases with SCV for policy Π_2. Policy Π_1, on the other hand, exhibits a U-shaped relationship between the SCV and project eNPV. In this particular instance, the eNPV of the project is largest when activity durations are highly uncertain (exponentially distributed). This contrasts with the intuition that an increase in uncertainty necessarily entails a decrease of system performance. Figure 4(c) depicts the eNPV of the *optimal* policy for each level of duration variability; here the U-shaped pattern is more pronounced. Again, we find that higher operational variability does not always lead to lower project values.

Ward and Chapman [49] argue that all current project risk-management processes induce a restricted focus on the management of project uncertainty. In part, this is because the term 'risk' encourages a 'threat' perspective: we refer the reader to the examples of risk events in the model for variability reduction by Ben-David and Raz [9] and Gerchack [21]. Ward and Chapman state that a focus on 'uncertainty' rather than risk could enhance project risk management, providing an important difference in perspective, including, but not limited to, an enhanced focus on opportunity management, an 'opportunity' being a 'potential welcome effect on project performance.' Ward and Chapman suggest that management strive for a shift from a threat focus towards greater concern with understanding and managing all sources of uncertainty, with both up-side and down-side consequences, and explore and understand the origins of uncertainty before seeking to manage it. They suggest using the term 'uncertainty management,' encompassing both 'risk management' and 'opportunity management.' The suggestions of Ward and Chapman are seconded from an operational viewpoint by our computational results. See also Loch et al. [32] for examples of how downside risks can sometimes be turned into upside opportunity (e.g., p. 5 and p. 20).

8 Summary and conclusions

In this article, we have developed a generic model for the optimal scheduling of R&D-project activities with stochastic durations, non-zero failure probabilities and multiple trials subject to precedence constraints. We assess the effect of different degrees of activity duration variability on the expected NPV of a project. We illustrate that higher operational variability does not always lead to lower project values, meaning that (sometimes costly) variance reduction strategies are not always advisable.

Appendix 1 Some technical results used in Section 5.1

Suppose that $X_i \sim \mathrm{Exp}(\lambda_i)$ for $i = 1, \ldots, \omega$, so the cdf of X_i is $F_{X_i}(t) = 1 - e^{-\lambda_i t}$, and that the X_i are independent. It then holds that $Y = \min_{i=1,\ldots,\omega}\{X_i\}$ is also exponential, with rate $\sum_i \lambda_i$. This can be seen as follows:

$$
\begin{aligned}
F_Y(t) = Pr[Y \le t] &= 1 - Pr[Y > t] \\
&= 1 - Pr[X_1 > t] \cdot \ldots \cdot Pr[X_\omega > t] \\
&= 1 - e^{-\lambda_1 t} \cdot \ldots \cdot e^{-\lambda_\omega t} \\
&= 1 - \exp\left(-t \sum_i \lambda_i\right),
\end{aligned}
$$

which we recognize as the cdf of an exponential variable with the mentioned rate.

The following is a classic result known as that of 'competing exponentials':

$$
Pr[k = \arg \min_{i=1,\ldots,\omega} \{X_i\}] = \frac{\lambda_k}{\sum_i \lambda_i}.
$$

For $\omega = 2$, we have

$$
\begin{aligned}
Pr[X_1 \le X_2] &= \int_0^\infty \int_{t_1}^\infty \lambda_1 e^{-\lambda_1 t_1} \lambda_2 e^{-\lambda_2 t_2} dt_2 dt_1 \\
&= \int_0^\infty \lambda_1 e^{-\lambda_1 t_1} e^{-\lambda_2 t_1} dt_1 \\
&= \lambda_1 \int_0^\infty e^{-(\lambda_1 + \lambda_2) t_1} dt_1 \\
&= \frac{\lambda_1}{\lambda_1 + \lambda_2} \int_0^\infty e^{-u} du = \frac{\lambda_1}{\lambda_1 + \lambda_2}.
\end{aligned}
$$

These integrals generalize straightforwardly to compute $Pr[(X_k \le X_1) \wedge (X_k \le X_2) \wedge \ldots]$.

In a very similar way, the appropriate discount factor in expectation, with continuous per-unit discount rate r over the timespan $Y = \min_{i=1,\ldots,\omega}\{X_i\}$, can be obtained as follows. We determine the present value e^{-rY} of a cash flow of 1 at time Y. Above, we have already established the distribution of Y. We obtain

$$
\int_0^\infty e^{-rt} \left(\sum_i \lambda_i\right) e^{-\left(\sum_i \lambda_i\right)t} dt = \ldots = \frac{\sum_i \lambda_i}{r + \sum_i \lambda_i}.
$$

One might argue that these computations are insufficient for application in Section 5 because the cash flow to be discounted actually depends on which of the X_i is the smallest. We now show that this does not preclude the foregoing simple result from being applied.

Define the function

$$\zeta(X_1, X_2, \ldots, X_\omega) =$$
$$\begin{cases} c_1 & \text{if } X_1 \leq X_2, X_1 \leq X_3, \ldots, \\ c_2 & \text{if } X_2 < X_1 \text{ and } X_2 \leq X_3, X_2 \leq X_4, \ldots, \\ \vdots & \\ c_\omega & \text{if } X_\omega < X_1, X_\omega < X_2, \ldots. \end{cases}$$

We wish to determine the expected *discounted* value of ζ. We find

$$\begin{aligned} &\int_0^\infty c_1 e^{-rt_1}.\lambda_1 e^{-\lambda_1 t_1}.e^{-\sum_{i\neq 1}\lambda_i t_1} dt_1 + \\ &\int_0^\infty c_2 e^{-rt_2}.\lambda_2 e^{-\lambda_2 t_2}.e^{-\sum_{i\neq 2}\lambda_i t_2} dt_2 + \\ &\ldots \\ &\int_0^\infty c_\omega e^{-rt_\omega}.\lambda_\omega e^{-\lambda_\omega t_\omega}.e^{-\sum_{i\neq \omega}\lambda_i t_\omega} dt_\omega \\ &= \frac{\lambda_1 c_1}{r + \sum_i \lambda_i} + \frac{\lambda_2 c_2}{r + \sum_i \lambda_i} + \ldots + \frac{\lambda_\omega c_\omega}{r + \sum_i \lambda_i} \\ &= \left(\frac{\sum_i \lambda_i}{r + \sum_i \lambda_i}\right)\left(\frac{\lambda_1}{\sum_i \lambda_i} c_1 + \ldots + \frac{\lambda_\omega}{\sum_i \lambda_i} c_\omega\right). \end{aligned}$$

The first term of the right-hand side of the final line of these equations is exactly the discount factor that we already obtained, while the second term is the expectation $\mathbb{E}[\zeta(X_1, \ldots, X_n)]$ of ζ *without discounting*. In other words, we can separate the computation of the expected value and the application of the discounting factor.

Appendix 2 Definition of a PH distribution

A PH distribution with parameter $(\boldsymbol{\tau}, \mathbf{T})$, denoted by PH$(\tau, \mathbf{T})$, is the distribution of the time until absorption into state 0 in a Markov chain on the states $\{0, 1, \ldots, z\}$ with initial probability vector $\boldsymbol{\tau} = \{\tau_0, \tau_1, \ldots, \tau_z\}$ and infinitesimal generator

$$\mathbf{Q} = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{t} & \mathbf{T} \end{bmatrix},$$

where $\mathbf{T}$ is the matrix containing the transition rates between transient states $\{1, \ldots, z\}$, $\mathbf{0}$ is a row vector of zeroes and $\mathbf{t} = -\mathbf{Te}$, where $\mathbf{e}$ is a column vector of ones. In addition, define the stochastic matrix $\mathbf{P}$ of the corresponding embedded discrete-time

Markov chain. Let π_{uv} and q_{uv} denote the v^{th} entry of the u^{th} row of the embedded Markov chain $\mathbf{P}$ and the infinitesimal generator $\mathbf{Q}$, respectively. The entries of $\mathbf{P}$ are computed as follows:

$$\pi_{uv} = \begin{cases} -\frac{q_{uv}}{q_{uu}} & \text{if } u > 0 \wedge u \neq v, \\ 1 & \text{if } u = 0 \wedge v = 0, \\ 0 & \text{otherwise.} \end{cases}$$

We will consider only acyclic PH distributions, in which each state is never visited more than once in the Markov chain whose absorption time defines the PH distribution. Examples of acyclic PH distributions include: the exponential, the Erlang, the hypo-exponential, the hyper-exponential and the Coxian distribution (Osogami [41]). Note that for acyclic PH distributions, it is possible to index the statespace such that $\mathbf{T}$ is an upper-triangular matrix.

In this article, we distinguish three PH distributions: (1) the exponential distribution; (2) the hypo-exponential distribution; and (3) the two-phase Coxian distribution. An exponential distribution with rate parameter λ is an acyclic PH distribution with initial probability vector $\boldsymbol{\tau} = \{0, 1\}$ and infinitesimal generator

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 \\ \lambda & -\lambda \end{bmatrix}.$$

The exponential distribution itself is the simplest of PH distributions. An Erlang distribution is the convolution of z exponentials that have identical rate parameter λ. The corresponding acyclic PH distribution has z phases, initial probability vector $\boldsymbol{\tau} = \{0, 1, 0, \ldots, 0\}$ and infinitesimal generator

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & -\lambda & \lambda & \ldots & 0 & 0 \\ 0 & 0 & -\lambda & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & -\lambda & \lambda \\ \lambda & 0 & 0 & \ldots & 0 & -\lambda \end{bmatrix}.$$

A hypo-exponential distribution or generalized Erlang distribution is the convolution of z exponential distributions with possibly different rate parameters. The corresponding acyclic PH distribution has z phases, initial probability vector $\boldsymbol{\tau} = \{0, 1, 0, \ldots, 0\}$ and infinitesimal generator

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & -\lambda_1 & \lambda_1 & \dots & 0 & 0 \\ 0 & 0 & -\lambda_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -\lambda_{z-1} & \lambda_{z-1} \\ \lambda_z & 0 & 0 & \dots & 0 & -\lambda_z \end{bmatrix}.$$

A two-phase Coxian distribution is a mixture of two exponential distributions and corresponds to an acyclic PH distribution that has initial probability vector $\boldsymbol{\tau} = \{0, 1, 0\}$ and infinitesimal generator

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 \\ (1-\beta)\lambda_1 & -\lambda_1 & \beta\lambda_1 \\ \lambda_2 & 0 & -\lambda_2 \end{bmatrix}.$$

A transition is made from the first phase towards the second phase with probability β. If the second phase is not visited (with probability $1-\beta$), absorption takes place after visiting the first phase.

Appendix 3 Moment matching

Let λ^{-1} and σ^2 denote the mean and variance of a random variable X, respectively. The squared coefficient of variation of X is $\varphi^2 = \sigma^2\lambda^2$. We describe the two-moment matching procedure described in Creemers et al. [15] that allows to determine the parameters of the PH distribution that matches the mean and the variance of X.

We distinguish three cases. The first case is $\varphi < 1$; then a hypo-exponential can be used to model X's distribution, with the following parameters:

$$\begin{aligned} z &= \lceil \varphi^{-2} \rceil, \\ \lambda_z &= \frac{1 + \sqrt{(z-1)(z\varphi^2 - 1)}}{\lambda(1 - z\varphi^2 + \varphi^2)}, \\ \lambda_u &= \frac{(z-1) - \sqrt{(z-1)(z\varphi^2 - 1)}}{\lambda(1-\varphi^2)}, \end{aligned}$$

for all $u \in \{1, 2, \dots, z-1\}$. The initial probability vector of the corresponding PH distribution is defined as follows:

$$\tau_u = \begin{cases} 1 & \text{if } u = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Alternatively, $\varphi > 1$ and X can be modeled via a two-phase Coxian distribution with parameters

$$\begin{aligned}
\tau_0 &= 0, \\
\tau_1 &= 1, \\
\tau_2 &= 0, \\
\beta &= \frac{2\left(\kappa - 1\right)^2}{1 + \varphi^2 - 2\kappa}, \\
\lambda_1 &= \frac{\lambda}{\kappa}, \\
\lambda_2 &= \frac{2\lambda\left(\kappa - 1\right)}{2\kappa - 1 - \varphi^2},
\end{aligned}$$

where κ is a value that can be chosen to determine the expected duration of the first phase. In this article we assume $\kappa = 0.5$.

The third case is $\varphi = 1$ and X is then approximated by an exponential distribution, so

$$\begin{aligned}
\tau_0 &= 0, \\
\tau_1 &= 1, \\
\lambda_1 &= \lambda.
\end{aligned}$$

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FACULTY OF ECONOMICS AND BUSINESS
Naamsestraat 69 bus 3500
3000 LEUVEN, BELGIË
tel. + 32 16 32 66 12
fax + 32 16 32 67 91
info@econ.kuleuven.be
www.econ.kuleuven.be